Mathematics Grade 7 Curriculum

Implemented September 2008

New Nouveau Brunswick

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Educational Programs and Services

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BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, the Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

GOALS FOR MATHEMATICALLY LITERATE STUDENTS

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

OPPORTUNITIES FOR SUCCESS

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices. Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals. Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

DIVERSE CULTURAL PERSPECTIVES

Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of all students.

Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components. Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. It is important to note that these general instructional strategies may not apply to all students.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

ADAPTING TO THE NEEDS OF ALL LEARNERS

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

CONNECTIONS ACROSS THE CURRICULUM

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

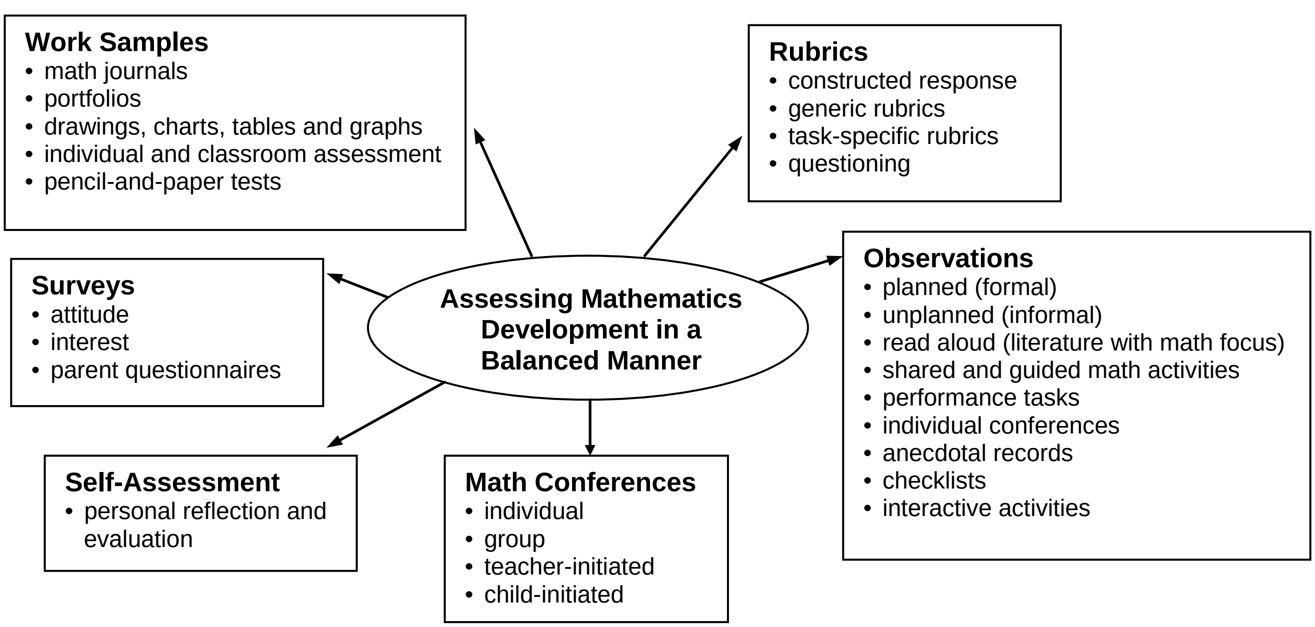
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

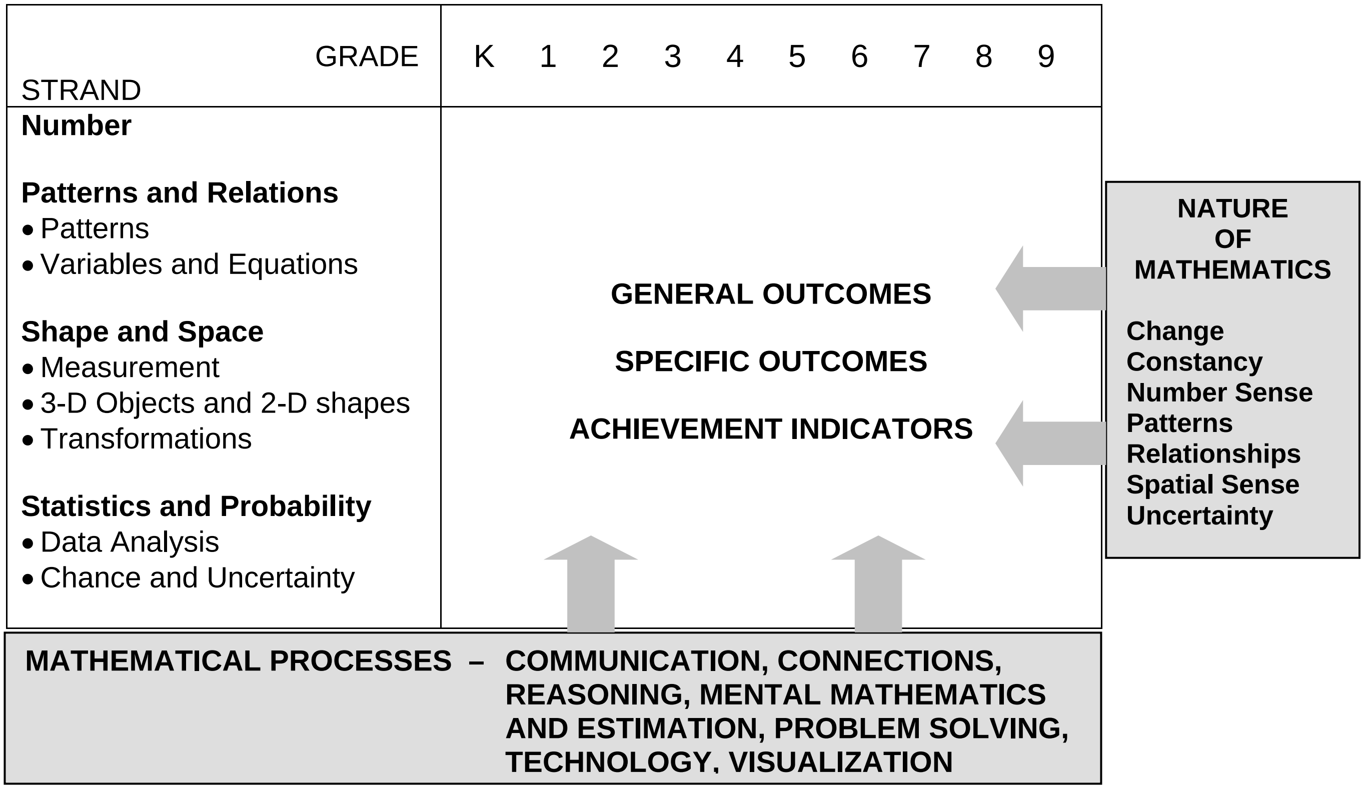
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR K – 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



INSTRUCTIONAL FOCUS

The New Brunswick Curriculum is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.
- There is a greater emphasis on mastery of specific curriculum outcomes.

The mathematics curriculum describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

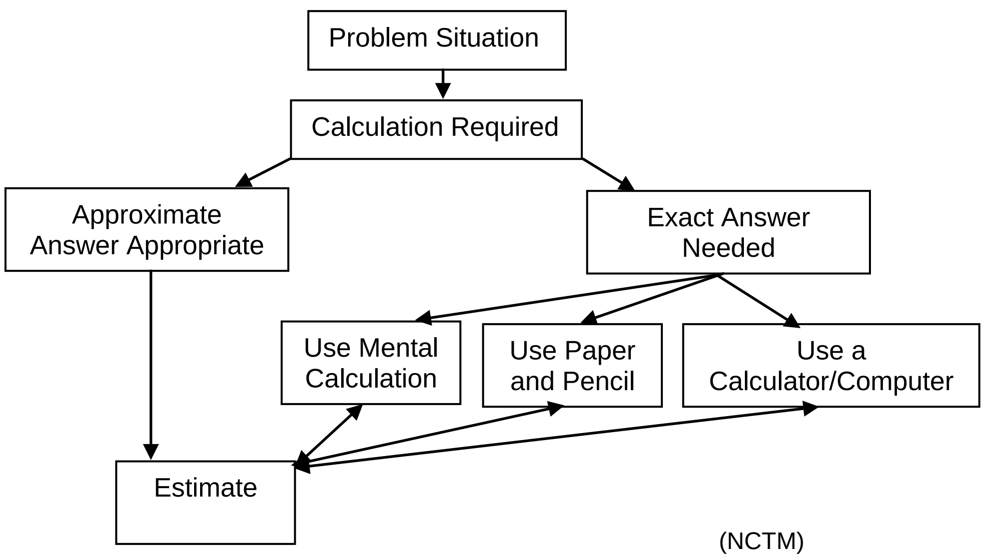
Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mental Mathematics and Estimation [ME]
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005).
Students proficient with mental mathematics “*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*” (Rubenstein, 2001). Mental mathematics “*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below.



(NCTM)

Problem Solving [PS]
Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “*How would you...?*” or “*How could you...?*” the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Technology [T]
Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.
Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]
Visualization "*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **relationships**, **patterns**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

STRUCTURE OF THE MATHEMATICS CURRICULUM

STRANDS

The learning outcomes in the New Brunswick Curriculum are organized into four strands across the grades, K–9. Strands are further subdivided into sub-strands which are the general curriculum outcomes.

OUTCOMES AND ACHIEVEMENT INDICATORS

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement Indicators are one example of a representative list of the depth, breadth and expectations for the outcome. Achievement indicators are pedagogy and context free.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems
	Variables and Equations: Represent algebraic expressions in multiple ways
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them
	Transformations: Describe and analyze position and motion of objects and shapes
Statistics and Probability (SP)	**Data Analysis**: Collect, display and analyze data to solve problems
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The specific curriculum outcomes (SCOs) are presented on individual four-page spreads as illustrated below.

GCO:
SCO: (specific curriculum outcome and mathematical processes)
Key for mathematical processes

Scope and Sequence
Current Grade

Elaboration
Guiding Questions

(Describes the "big ideas" and what the students should learn this year in regards to this concept.)

Page 1

GCO:
SCO:

Achievement Indicators

Guiding Questions

(Describes what could be observed to determine whether students have met the specific outcome.)

Page 2

GCO:
SCO:

Planning for Instruction
Guiding Questions

Choosing Instructional Strategies
(Lists general strategies to assist in teaching this outcome.)

Suggested Activities
(Lists possible specific activities to assist students in learning this concept.)

Possible Models

Page 3

GCO:
SCO:

Assessment Strategies
Guiding Questions

(Overview of assessment)

Whole Class/Group/Individual Assessment
(Lists sample assessment tasks.)

Follow-up on Assessment
Guiding Questions

Page 4

SCO: **N1: Determine and apply the divisibility rules for 2, 3, 4, 5, 6, 8, 9 or 10, and explain why a number cannot be divided by 0.** [C, R]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N3 Demonstrate an understanding of factors and multiples by: determining multiples and factors of numbers less than 100; identifying prime and composite numbers; solving problems involving multiples.	**N1** Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.	**N1** Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). **N2** Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Exploration of the **divisibility** rules serves as an excellent opportunity to extend number sense. Knowledge of divisibility rules will provide a valuable tool for mental arithmetic and general development of operation sense.

Students should be reminded of the divisibility rules for 2, 5, and 10 which most should recall readily. Once students understand divisibility for 2 and 3, they can use this knowledge to develop a means of testing for divisibility by 6. This should be seen as a problem solving opportunity for students. They can also explore whether this strategy will always work for other numbers such as 8 and 10

The divisibility rules are as follows (this is a suggested order for instruction):
A number is divisible by

- 2 if it is even
- 5 if it ends in a 5 or a 0
- 10 if it ends in a 0
- 3 if the sum of the digits is divisible by 3
- 6 if the number is divisible by 3 and even
- 9 if the sum of the digits is divisible by 9
- 4 if the number formed by the last two digits is divisible by 4
- 8 if the number is divisible by 4 and the resulting quotient is even (for 92, think $92 \div 4 = 23$, since 23 is not even, 92 is not divisible by 8); or if the number represented by the last 3 digits is divisible by 8

To avoid an arbitrary rule for not being able to divide by 0, use a repeated subtraction meaning for division. For example, $20 \div 5$, you can subtract 5 four times from 20 until you get to 0, so $20 \div 5 = 4$. So, for $6 \div 0$, ask how many times can you subtract 0 from 6 before you get to 0? There is no answer; you will never get to 0 ($6 - 0 - 0 - 0 = 6$).

SCO: **N1: Determine and apply the divisibility rules for 2, 3, 4, 5, 6, 8, 9 or 10, and explain why a number cannot be divided by 0.**
[C, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10.
- Determine the factors of a given number using the divisibility rules.
- Explain, using an example, why numbers cannot be divided by 0.

SCO: **N1: Determine and apply the divisibility rules for 2, 3, 4, 5, 6, 8, 9 or 10, and explain why a number cannot be divided by 0.**
[C, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

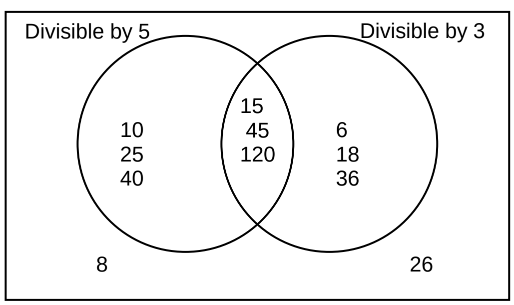
Consider the following strategies when planning lessons:

- Organize instruction so that the students develop the divisibility rules themselves through investigations.
- Explore why some divisibility rules work, such as the divisibility test for 4.
- Consider 2346. 2346 = 2300 + 46. Since 100 is divisible by 4, all multiples of 100 are divisible by 4; therefore, 2300 is divisible by 4. All that remains is to determine if 46 is divisible by 4.
- Use a 100 chart to explore patterns of multiples.
- Explore the use of a calculator as a tool to test for divisibility. Students should realize that the test for divisibility on a calculator involves dividing to see if the quotient is a whole number, not a decimal.

Suggested Activities

- Have students explore divisibility rules for 3, 6, and 9. Ask them to write the first 10 multiples of 3. Ask what they notice about the numbers. If no student mentions the sum of the digits, ask them to find the sum of the digits and describe what they notice. Ask them which numbers in the same list are divisible by 6. Ask what they notice about these numbers. Test the conclusions, using numbers such as 393, 504, and 5832.
- Sort a given set of numbers based upon their divisibility using organizers, such as Venn and Carroll diagrams.
- Create a Carroll diagram or a Venn diagram to sort the following numbers based on the divisibility rules for 3 and 5: 6, 8, 10, 15, 18, 25, 26, 36, 40, 45, 120.

Carroll Diagram	Divisible by 3	Not Divisible by 3
Divisible by 5	15 45 120	10 25 40
Not Divisible by 5	6 36 18	8 26



Extension: Which numbers are also divisible by 15?

Possible Models: calculators, hundred chart, Venn diagram, Carroll diagram

SCO: **N1: Determine and apply the divisibility rules for 2, 3, 4, 5, 6, 8, 9 or 10, and explain why a number cannot be divided by 0.**
[C, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Show numbers at random on the “board”. Ask students to determine by what numbers they are divisible.
- Have students create a 3 digit number that is divisible by both 4 and 5. Is it also divisible by 2 and 8?
- Ask students to complete the number by filling in each blank with a digit. Ask them to explain how they know their answer is correct.
  a. 26_ is divisible by 10
  b. 154_ is divisible by 2
  c. _6_ is divisible by 6
  d. 26_ is divisible by 3
  e. 1_ 2 is divisible by 9
  f. 15_ is divisible by 4

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N2: Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.**
[ME, PS, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N1 Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth **N8** Demonstrate an understanding of multiplication and division of decimals. **N9** Explain and apply the order of operations, excluding exponents, with and without technology	**N2** Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.	

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should know when it is appropriate to use a paper-pencil algorithm, a mental procedure, or a calculator for the mathematical operations involving whole numbers and/or decimals.
The students need to understand the relationship between whole number and decimal number operations, including order of operations begun in grade 6. Emphasis should be placed on place value and estimation ideas and ensure that instruction does not focus on students simply mastering procedural rules without a conceptual understanding. It is important that a problem solving context is used to help ensure the relevance of the operations.

Addition and subtraction questions should also be presented horizontally, as well as vertically, to encourage alternative computational strategies. Students should be able to use algorithms of choice when they calculate with pencil-and-paper methods. It is important that the algorithms developed by students are respected, however, if their strategies are inefficient, students should be guided toward more appropriate ones. For example, when adding numbers such as 4.2 and 0.23, students should be encouraged to use a “front-end” approach and add the whole numbers, tenths, and hundredths.

Estimation should be used to develop a sense of the size of the answer for all calculations involving decimals. For example, one might round each of the decimal numbers 2.8 x 8.3 for an estimate of 24 (3 x 8). When estimation is an automatic response students will, when faced with a calculation, not depend on recalling the “counting back decimal places,” rule.

Multiplication and division of two numbers will produce the same digits, regardless of the position of the decimal point. As result, for most practical purposes, there is no reason to develop new rules for decimal multiplication and division. Rather, the computations can be performed as whole numbers with the decimal placed by way of estimation (Van de Walle & Lovin, vol. 3, 2006, p. 107).

SCO: **N2: Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.**
[ME, PS, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Solve a given problem involving the addition of two or more decimal numbers.
- Solve a given problem involving the subtraction of decimal numbers.
- Solve a given problem involving the multiplication of decimal numbers.
- Solve a given problem involving the multiplication or division of decimal numbers with 2-digit multipliers or 1-digit divisors (whole numbers or decimals) without the use of technology.
- Solve a given problem involving the multiplication or division of decimal numbers with more than a 2-digit multiplier or 1-digit divisor (whole number or decimal), with the use of technology.
- Place the decimal in a sum or difference using front-end estimation, e.g., for 4.5 + 0.73 + 256.458, think 4 + 256, so the sum is greater than 260.
- Place the decimal in a product using front-end estimation, e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24.
- Place the decimal in a quotient using front-end estimation, e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m.
- Check the reasonableness of solutions using estimation.
- Solve a given problem that involves operations on decimals (limited to thousandths) taking into consideration the order of operations.

SCO: **N2: Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.**
[ME, PS, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use patterns to help students understand the placement of the decimal in the product of two decimal amounts. For example, 9 x 7 = 63 therefore, 9 x 0.7 (or 7 tenths) = 6.3 or 63 tenths.
- Use the "area model" both used concretely (base ten blocks) and pictorially (grid paper or array). When considering multiplication by a decimal, students, should recognize that, for example, 0.8 of something will be almost that amount, but not quite, and 2.4 multiplied by an amount will be double the amount with almost another half of it added on.
- Use story problems to provide students with a relevant context for completing computations.
- Focus on strategies such as rounding and front-end estimation. For example:
  Rounding: 789.6 ÷ 89, think: "90 multiplied by what number would give an answer close to 800?"
  Front-end estimation: 6.1 x 23.4 might be considered to be 6 x 20 (120) plus 6 x 3 (18) plus a little more for an estimate of 140, or 6 x 25 = 150.

Suggested Activities

- Ask students to describe how to calculate 3 x 1.25 by thinking of it as money.
- Have students describe the situation by referring to money: 2.40 ÷ 0.1 = 24 (24 dimes in $2.40).
- Ask students to work in pairs sharing strategies for estimating in situations such as:
  - 6.1 m of material at $4.95 a metre;
  - area of a rectangular plot of land 24.78 m x 9.2 m;
  - 0.5 of a length of rope 20.6 m long;
  - 9.7 kg of beef at $4.59/kg;
  - 4.38 kg of fish at $12.59/kg.
- Provide students with a variety of division questions that result in a remainder and have them investigate and discuss the meaning of the remainders.
- Have students display the multiplication of two factors using an open array as shown in the example below. To find the overall product, add the partial products in the array. The numbers used on the outside of the array can be partitioned in flexible ways to create "nice" numbers to multiply.

2.4 × 3.7

	2	0.4
3	6	1.2
0.7	1.4	0.28

Possible Models: base ten blocks, number lines, metre sticks, place value chart, money, calculator, area models, grid paper, open array

SCO: **N2: Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.**
[ME, PS, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

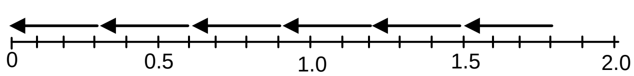
Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask the student to create addition, subtraction, multiplication and division word problems, each with an answer of 4.2.
- Ask the student to draw or build a model to illustrate 4 x 3.45 and a model to show how to find 5.28 ÷ 4.
- Ask the students to find the missing digits:
  5.☐3
  X ☐
  3☐.58
- Ask students how are the results of 423 ÷ 3 and 42.3 ÷ 3 related?
- Ask students to use a model to show why 4.2 ÷ 0.2 is the same as 42 ÷ 2.
- Ask why someone might find it easier to divide 8.8 by 0.2 than 1.1 by 0.3?
- Ask the student to respond to the following: Jade said, 3.45 x 4 must be 1.380. There is only one digit before the decimal place in 3.45, so there must be one digit before the decimal place in the product.
- Tell the students that two decimals are multiplied. The product is 0.48. Ask: What might they have been? Give two other pairs of factors.
- Ask the student to explain how the diagram shows that 1.8 ÷ 0.3 = 6



- Have the student draw a quadrilateral with a perimeter of 16.3 cm, where no sides are whole numbers.
- Ask students what the remainder means when you divide 4.1 by 4?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N3: Solve problems involving percents from 1% to 100%.** [C, CN, PS, R, T, ME]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N5 Demonstrate an understanding of ratio, concretely, pictorially and symbolically. **N6** Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.	**N3** Solve problems involving percents from 1% to 100%.	**N3** Demonstrate an understanding of percents greater than or equal to 0%. **N4** Demonstrate an understanding of ratio and rate. **N5** Solve problems that involve rates, ratios and proportional reasoning.

ELABORATION

Guiding Questions:

• What do I want my students to learn?

• What do I want my students to understand and be able to do?

Percents are simply hundredths, and as such they should be introduced as a third way of writing both fractions and decimals. Number sense for percent should be developed through the use of **benchmarks**:

- 100% is all
- 50% is half
- 25% is a quarter
- 10% is a tenth
- 1% is one hundredth

Students should be able to flexibly move between **percent**, **fraction** and **decimal equivalents** in problem solving situations. For example, when finding 25% of a number, it is often much easier to use $\frac{1}{4}$ and then divide by 4 as a means of finding or estimating the percent. Students should make immediate connections between other percentages and their fraction equivalents, such as 50%, 75%, $33\frac{1}{3}$% and 20%, 30%, 40%, etc. Encourage students to recognize that percents such as 51% and 12% are close to benchmarks, which could be used for estimation purposes. Students should be able to calculate 1%, 5% (half of 10%), 10%, and 50% mentally using their knowledge of benchmarks. When exact answers are required, students should be able to employ a variety of strategies in calculating percent of a number. Students should be able to solve problems which involve finding a, b, or c in the relationship a% of b = c, using estimation and calculation.

Discussion should also focus on the contexts in which 1% would be considered high and the contexts in which 90% would be considered low. Everything is relative to the size of the whole.

The conceptual understanding developed for this outcome should flow from meaningful problem solving contexts.

SCO: **N3: Solve problems involving percents from 1% to 100%.**
[C, CN, PS, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Express a given percent as a decimal or fraction.
- Solve a given problem that involves finding a percent.
- Determine the answer to a given percent problem where the answer requires rounding and explain why an approximate answer is needed, e.g., problem situation involving money.

SCO: **N3: Solve problems involving percents from 1% to 100%.**
[C, CN, PS, R, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

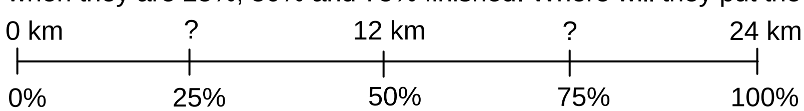
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Employ a variety of strategies when exact answers are required to calculate the percent of a number:
  - changing percent to a decimal and multiplying
    12% of 80 = 0.12 × 80 (9.6)
  - finding 1% and then multiplying
    12% of 80, find 1% of 80 = 0.8 × 12 (9.6)
  - changing to a fraction and dividing
    25% of 60 = $\frac{1}{4}$ × 60 (60 ÷ 4)
  - calculating proportions
    12% of 80 → $\frac{12}{100} = \frac{?}{80}$
- Provide a 10 × 10 grid so students have a visual image of the 1% method. To find 6% of 400, tell students you have $400 and you want to share it equally among the 100 cells. Ask them how much would be in each cell? In 2 cells? In 6 cells? Students can also use this method to estimate; for example, they can estimate 8% of 619 by first mentally finding 8% of 600.
- Have students create problems that utilize percent. They can be given flyers from local supermarkets and/or department stores and use these to create problems which involve calculating the total savings when certain items are purchased at the sale price.
- Use a double number line as a useful tool for understanding percentage. For example: During a 24 km walk/run to raise money for a local children's hospital, organizers would like to put up markers to tell participants when they are 25%, 50% and 75% finished. Where will they put the signs?



Suggested Activities

- Have students solve problems such as the following.
  The manager of a concert hall indicated that, in order to make a profit, the hall must be filled to at least 70% capacity or else the price of each ticket will need to increase. The seating capacity is 1200, and advance ticket sales are at 912. Have enough tickets been sold so that ticket prices will remain low?
- Describe more than one method that could be used to mentally estimate 22% of 310. How could you find the exact answer by calculating mentally?
- Ask students to explain why this works: Zack found 52% by finding 50% + 1% + 1%. Have them use this strategy to find 52% of 160.

Possible Models: calculator, hundredths circle, hundred grid, double number line

SCO: **N3: Solve problems involving percents from 1% to 100%.**
[C, CN, PS, R, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Tell students that a jacket is now selling for $64. The sign above it indicates that the price was reduced by 20%. What was the original selling price?
- Ask students, if 30 is close to 80% of a number, what do you know about the number?
- Ask students, if 60% a good estimate for $\frac{30}{70}$? Have them explain their reasoning.
- Ask students to explain how they would estimate 48% of something.
- Ask students, if 2% of a certain number is 0.46, what would 10% of the number be? What is the number?
- Ask students, what percentage of the total arena capacity was used if $\frac{7}{8}$ of the tickets were sold for a concert?
- Ask students to:
  - Explain why 70% is not a good estimate for 35 out of 80.
  - Explain how to estimate the percentage when a test score is 26 correct out of 55.
  - Change each of the following to a percent mentally and explain their thinking:
    $\frac{2}{5}, \frac{4}{25}, \frac{6}{50}, \frac{8}{20}$.
  - Estimate the percent for each of the following and explain their thinking:
    $\frac{7}{48}, \frac{5}{19}, \frac{7}{20}$.
  - Indicate what percent of a book is left to read after 60 out of 150 pages have been read.
  Have students explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N4: Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.**
[C, CN, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N1 Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth. **N4** Relate improper fractions to mixed numbers. **N6** Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.	**N4** Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.	

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Decimal numbers are simply another way of writing fractions. Maximum flexibility is gained by understanding how the two systems are related (Van de Walle & Lovin, vol. 3, 2006, p. 107).

Decimals and proper fractions are both parts of wholes. All fractions can be expressed as **terminating or repeating decimals** and vice versa. A few students will already know the decimal equivalents of some simple fractions (e.g., $\frac{1}{2} = 0.5$, $\frac{1}{4} = 0.25$, $\frac{1}{5} = 0.2$) as well as any fraction with a denominator of 10, 100, or 1000. For example, to locate 0.75 on a number line, many students think of 0.75 as being three quarters of the way from 0 to 1. Many students, however, believe that the only fractions which can be described by decimals are those with denominators which are a power of 10 or a factor of a power of 10. By building on the connection between fractions and division, students should be able to represent any fraction in decimal form, using the calculator as an aid.

Many fractional numbers produce decimals that will not **terminate**, but produce repeating patterns such as thirds and ninths. Students should be introduced to the terminology "**repeating**" and "**period**" as well as **bar notation** used to indicate repeating periods. A bar is drawn over the digits that repeat. The patterns produced by fractions with a variety of denominators should be explored since many have particularly interesting periods.

Students should use calculators when appropriate to find the decimal form for some fractions and predict the decimal for other fractions. Students should also be aware of the effect of calculator rounding (i.e., automatic rounding caused by the limit on the number of digits which the calculator can display). Students should use their knowledge of the patterns to determine the fractional form of repeating decimals.

SCO: **N4: Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.**
[C, CN, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Predict the decimal representation of a given fraction using patterns, e.g.,

$$\frac{1}{11} = 0.\overline{09}, \quad \frac{2}{11} = 0.\overline{18}, \quad \frac{3}{11} = ?$$

- Match a given set of fractions to their decimal representations.
- Sort a given set of fractions as repeating or terminating decimals.
- Express a given fraction as a terminating or repeating decimal.
- Express a given repeating decimal as a fraction.
- Express a given terminating decimal as a fraction.
- Provide an example where the decimal representation of a fraction is an approximation of its exact value. (For example, two thirds equals approximately 0.67)

SCO: **N4: Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.**
[C, CN, R, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use both proper fractions and mixed numbers in activities.
- Explore patterns of different fraction families (e.g., $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}$) with a calculator.
- Have students recognize that only fractions that can be expressed with base ten denominators (10, 100, and 1000) will be terminating when written in decimal form. For example:
  $3\frac{2}{5} = 3\frac{4}{10} = 3.4$ $2\frac{3}{8} = 2\frac{375}{100} = 2.375$
- Provide frequent opportunities to read terminating decimals (e.g., 0.312, is read as three hundred twelve thousandths). When a student reads a terminating decimal, it should be clear how to write it in fractional form.

Suggested Activities

- Give students a set of fractions such as $\frac{1}{13}, \frac{2}{13}, \frac{3}{13}$. Ask them to find a pattern and then use the pattern to predict the decimal for other fractions such as $\frac{4}{13}, \frac{5}{13}, \frac{10}{13}$.
- Ask students to compare the decimals using a calculator for the following pairs and have them discuss the similarities and differences they observe.
  a. $\frac{1}{12}$ and $\frac{1}{120}$ b. $\frac{3}{8}$ and $\frac{3}{80}$
  c. Since the decimal for $\frac{3}{16}$ is 0.1875, ask students to predict the fraction that would produce a decimal of 0.01875.
- Use base ten blocks to explain the decimal equivalents to fractions, even when these decimals repeat. For example, 1 ÷ 3 could be modeled by having 1 block shared by 3 people and decide how to share the remaining piece(s). Decimal squares can be used similarly by having students shade, for example, one third of the square and decide how to shade the remaining square(s).
- Tell students that a certain candy bar can easily be broken into 8 equal square pieces. There are 27 students in Suri's class, and she has $3\frac{1}{2}$ candy bars. Suri found how many eighths there were in the bars, using equivalent fractions. Are there enough pieces for each student to have a square of the candy? Explain how the answer was found. Represent as a decimal the fractional part that is left over.

Possible Models: base ten blocks, calculator, Decimal squares®, number line, fraction pieces, hundredth circle

SCO: **N4: Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.**
[C, CN, R, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students find the following decimal representations of fractions: $\frac{1}{11}$, $\frac{2}{11}$, $\frac{3}{11}$ and then ask students to:
  a. Predict the decimals for $\frac{5}{11}$ and $\frac{9}{11}$.
  b. Predict the fraction which will have 0.636363... as a decimal.
  c. Predict what the decimal for $\frac{8}{11}$ would look like on a calculator display if the calculator is set to display 8 places after the decimal.
  d. predict the fraction which will have 0.909090… as a decimal.
- Ask students if the fraction $\frac{1}{27}$ produces a repeating pattern.
- Ask: How does knowing that $\frac{1}{4}$ = 0.25 help you find the decimal form of $\frac{3}{4}$ and $\frac{5}{4}$?
- Tell students that Chris had a calculator which displayed 2.3737374. Chris concluded that it was not a repeating decimal. Ask students to explain why Chris drew this conclusion and whether or not they believe it to be a correct conclusion.
- Ask students which is larger 0.7 or $0.\overline{7}$? Have students explain their reasoning.
- Have the student describe a fraction which is a bit less than 0.4 and to justify the selection. Ask the student if he/she can name another fraction that is between these two?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N5: Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).**
[C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N3 Demonstrate an understanding of factors and multiples by: determining multiples and factors of numbers less than 100; identifying prime and composite numbers; solving problems involving multiples. **N4** Relate improper fractions to mixed numbers.	**N5** Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).	**N6** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

It is important to give students ample opportunity to develop strong fraction number sense and not immediately focus on common denominators and other rules of computation (Van de Walle & Lovin, vol. 3, 2006, p. 87). They must first explore the meaning of fractions, using different **models**: **region**, **set** and **length** or **measurement**. To help students add and subtract fractions correctly, and with understanding, teachers must help them develop an understanding of the **numerator** and **denominator**, **equivalence** and the relation between **mixed numbers** and **improper fractions** (NCTM, 2000, p. 218).

- The meanings of each operation with fractions are the same as the meanings for the operations of whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts. For addition and subtraction, it is critical to understand that the numerator tells the number of parts and the denominator the type of part.
- Estimation of fraction computations is tied almost entirely to concepts of the operations and of fractions. Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected size of the results (Van de Walle & Lovin, vol. 3, 2006, p. 66).

Developing **benchmarks** of $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, 1, etc., should be a focus of the **estimation** of **sums** and **differences** of fractions. It is necessary to encourage flexibility in thinking and provide learning opportunities in connecting:

- operations with whole numbers to operations with fractions;
- subtraction of fractions to addition of fractions;
- concrete, pictorial and symbolic representations;
- operations with fractions to real world problems (Alberta Education, 2004).

Once students have explored addition and subtraction of fractions with models and pictorially, they should represent these computations symbolically and explore algorithms. It is important for students to make the connection between addition and subtraction of fractions. Students should apply prior knowledge of factors to assist them in determining common denominators and simplifying fractions and mixed numbers.

SCO: **N5: Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).**
[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Use benchmarks to estimate the sum or difference of positive fractions or mixed numbers.
- Model addition and subtraction of given positive fractions or mixed numbers, using concrete representations, and record symbolically.
- Determine the sum of two given positive fractions or mixed numbers with like denominators.
- Determine the difference of two given positive fractions or mixed numbers with like denominators.
- Determine a common denominator for a given set of positive fractions or mixed numbers.
- Determine the sum of two given positive fractions or mixed numbers with unlike denominators.
- Determine the difference of two given positive fractions or mixed numbers with unlike denominators.
- Simplify a given positive fraction or mixed number by identifying a common factor (GCF) between the numerator and the denominator.
- Simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers.
- Solve a given problem involving the addition or subtraction of positive fractions or mixed numbers and determine if the solution is reasonable.

SCO: **N5: Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).**
[C, CN, ME, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

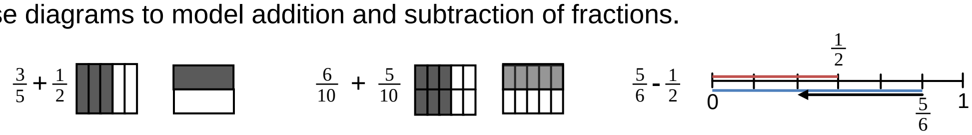
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Access prior knowledge of fractions from grades 5 and 6.
- Use concrete models to demonstrate the meaning of fractions. The numerator of a fraction counts or tells how many of the fractional parts (of the type indicated by the denominator) are under consideration. The denominator names the kind of fractional part that is under consideration (what is being counted). Students find it challenging to conceptualize what represents the "whole" so it is important to use a variety of materials so their understanding is not associated with a single model.
- Use a problem-solving context that is relevant to students.
- Connect problems applying the addition and subtraction of fractions and mixed numbers to similar problems with whole numbers. Include various structures of problems for addition and subtraction such as, part-part-whole and comparison from previous grades, such as $\frac{1}{2} + \square = \frac{5}{8}$ or $\square + \frac{1}{4} = \frac{2}{3}$.
- Connect the subtraction of fractions to the addition of fractions.
- Estimate sums and differences of fractions before calculating by using benchmarks (e.g., $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, 1).
- Have students explore sums and differences of fractions by using a variety of models.
- Emphasize that students connect the concrete, pictorial and symbolic representations for sums and differences of fractions. Once students internalize the fact that fractions can be added and subtracted symbolically, they become less reliant on concrete and pictorial models.
- Ensure that students record all solutions to fraction computations in the simplest form.

Suggested Activities

- Use concrete models and drawings to represent addition and subtraction questions. Have students solve contextual problems using the models. As they develop an understanding, students can record the steps symbolically as they solve the operations. Provide a variety of contexts: baking, mowing lawns, time, etc.
- Tell students that Jody added fractions and found the answer to be $\frac{5}{8}$. What could the fractions have been? How many different answers could there be?
- Use diagrams to model addition and subtraction of fractions.



Possible Models: fraction pieces and blocks, pattern blocks, number lines, counters, Cuisenaire rods®

SCO: **N5: Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).**
[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group Assessment/Individual Assessment

Provide a variety of models for students to use during assessments as needed.
Instruct students to show all their work.

- Have students explain why this doesn't make sense: Sam wrote $\frac{3}{4}$ hr + $\frac{1}{2}$ hr = $\frac{4}{6}$ hr., saying that he had worked on the computer for 45 minutes and watched television for half an hour. Explain his mistake in determining the total time he spent on these activities. What is the correct answer (expressed as a fraction)?
- Create three addition and three subtraction sentences that would have an answer of $\frac{3}{4}$.
- Ask students to answer the following and justify their responses.
  a) if an answer can be sixths when you add fourths and thirds.
  b) if an answer can be sevenths when you add fourths and thirds.
- Tell students that a container is half full. When half a cup of juice is added to it, the container is three quarters full. How much liquid can the container hold? Model or draw your answer.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N6: Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N7 Demonstrate an understanding of integers, concretely, pictorially and symbolically.	**N6** Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.	**N7** Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students develop understanding of operations with integers when emphasis is placed on the following:

Real world contexts
In everyday life there are many uses of integers, so **problem solving** plays a major role in developing understanding of integer operations. Students should see a connection between integers and their world around them through the use of problems using real-life contexts such as height above and below sea level, temperature, and banking (deposits and withdrawals), etc.

Relating operations with whole numbers to operations with integers
The set of integers is an extension of the whole number system to include the **opposite** of every whole number. Operations with integers build on operations with whole numbers. Since integers include the whole numbers and their opposites it can be said that integers are numbers that deal with opposites (direction) as well as quantity (magnitude).

Creating concrete, pictorial and symbolic representations
The two models most commonly used for solving **addition** and **subtraction** with integers are: two coloured counters and number lines. Both models depict the concepts of **quantity** and opposite and students should be given experiences with each. Quantity is signified by the number of counters or length of the arrows. Opposite is represented as different colours or different directions.

Knowing and using the zero principle
Emphasis must be placed on the **zero principle** and its application in addition and subtraction situations. The balance of positive and negative values is known as the zero principle. For example, (-1) + (+1) = 0, (-3) + (+3) = 0, (-17) + (+17) = 0. The sum of any opposite integers is zero. Based on this principle, zero and other integers can be expressed in multiple ways.

Connecting subtraction of integers to addition of integers
Students should develop an understanding why any subtraction number sentence can be written as an **equivalent** addition number sentence.

SCO: **N6: Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero.
- Illustrate, using a number line, the results of adding or subtracting negative and positive integers; e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position.
- Add two given integers, using concrete materials or pictorial representations, and record the process symbolically.
- Subtract two given integers, using concrete materials or pictorial representations, and record the process symbolically.
- Illustrate the relations between adding integers and subtracting integers.
- Solve a given problem involving the addition and subtraction of integers.

SCO: **N6: Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Emphasize that students begin with concrete to pictorial and finally to symbolic representations for sums and differences of integers.
- Connect the subtraction of integers to the addition of integers.
- Use problem-solving contexts which are relevant to students.
- Connect problems applying addition and subtraction of integers to similar problems with whole numbers.
- Have students explore sums and differences of integers by using a variety of models, such as algebra tiles and arrows on a number line. It should be noted that it is important to always use a key in pictorial representations of integers, so students know which colour represents positive integers and which represent negatives. There is no set standard for these and students should be flexible in their thinking.
- Include various structures of problems such as, “part-part-whole” and “comparison” from previous grades. For example, -6 - ☐ = 3; ☐ + 2 = -5; -9 = -4 + ☐.
- Have students justify the strategies they use in finding sums and differences of integers and provide opportunities to discuss strategies used by others.

Suggested Activities

- Ask students to solve addition and subtraction of integers through the use of Magic Squares.

	-7	
		-11
-9	-1	-2

- Have students roll two dice of different colours. Assign negative to one colour and positive to the other, and find the sum of the numbers rolled. Have them roll the two dice again, find the sum and add the result to their previous score. Have them exchange turns until one person reaches +20 or -20. Ask why it would be fair to accept +20 or -20 as the winning score.
- Ask students to solve and create problems using real-life situations such as: time zones, temperature, heights above and below sea level, profits and losses, games, sports, shares of stocks, etc.

Possible Models: two colour counters, number lines (vertical and horizontal), algebra tiles, thermometers

SCO: **N6: Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students use a number line or two colour counters to explain why the following calculations are correct.

  $-3 + 8 = 5$
  $-5 - 3 = -8$
  $-4 - (-6) = 2$
  $9 + (-2) = 7$
  $6 - 4 = 2$
  $8 - (-3) = 11$
- Tell students: Jon saved \$50 during the fall. He owes \$15 to his friend. He earned \$20 mowing lawns. What is Jon's net worth?
- Ask: Can you model +2 with an odd number of counters? Explain why or why not.
- Ask students to model (- 2) – (- 4) using a number line.
- Ask students if the sum of a negative integer and a positive integer is always negative. Explain why or why not.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N7: Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:** **• benchmarks** **• place value** **• equivalent fractions and/or decimals.** [CN, R, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
N1 Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth. **N4** Relate improper fractions to mixed numbers.	**N7** Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: • benchmarks • place value • equivalent fractions and/or decimals.	

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should continue to use conceptual methods to **compare fractions** and **decimals**, such as context problems and models. Students tend to think of fractions as sets or regions whereas they think of decimals as being more like whole numbers. A significant goal of instruction in decimal and fraction numeration should be to help students see that both systems represent the same concepts. For this reason it is important to use a variety of **models** and **benchmarks**. Money should not be used exclusively as a model for decimals as it is very limiting (typically only extends to hundredths).

Students need experiences comparing fractions with the same denominator, with the same numerator, and with unlike denominators. Students should develop a variety of strategies to compare fractions. They should also be able to identify fractions that are between any two given fractions.

A rich understanding of **place value** allows students to compare and order decimals using strategies similar to those used with whole numbers. Students should be discouraged from using the strategy of "adding zeros to a number" to create decimals of equal length without having first developed the conceptual understanding of place value as it relates to decimals.

Once students develop a sense of the benchmark fractions (e.g., $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$) and know the **decimal equivalents** (0.25, 0.5, 0.75), they are able to use them interchangeably as a powerful strategy for comparing and ordering fractions and decimals.

Decimal numbers are simply another way of writing fractions. Both notations have value. Maximum flexibility is gained by understanding how the two symbols are related (Van de Walle & Lovin, vol. 3, 2006, p. 107).

SCO: **N7: Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:**
- **benchmarks**
- **place value**
- **equivalent fractions and/or decimals.**

[CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Order the numbers of a given set that includes positive fractions, positive decimals and/or whole numbers in ascending or descending order, and verify the result using a variety of strategies.
- Identify a number that would be between two given numbers in an ordered sequence or on a number line.
- Identify incorrectly placed numbers in an ordered sequence or on a number line.
- Position fractions with like and unlike denominators from a given set on a number line and explain strategies used to determine order.
- Order the numbers of a given set by placing them on a number line that contains benchmarks, such as 0 and 1 or 0 and 5.
- Position a given set of positive fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position.
- Position a given set of positive decimals on a number line and explain strategies used to determine position.

SCO: **N7: Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:**
- **benchmarks**
- **place value**
- **equivalent fractions and/or decimals.**

[CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

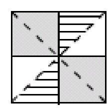
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

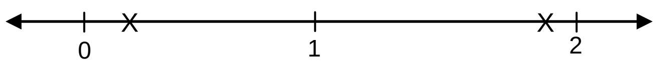
- Have students initially construct models and drawings (e.g., hundredths grid and number lines) to compare decimals before moving to comparing them with other strategies.
- Encourage students to compare fractions greater than one by considering them as mixed numbers. For example, which is greater, $\frac{10}{8}$ or $\frac{7}{5}$? A possible answer: "I know $\frac{7}{5}$ is greater because $\frac{10}{8}$ is $1\frac{2}{8}$, and $\frac{7}{5}$ is $1\frac{2}{5}$, and I know that $\frac{2}{5}$ is greater than $\frac{2}{8}$."
- Have students choose the greater fraction or decimal in a given a pair. Have them defend their choice. They must then prove their answer is correct using a model of their choice.

Suggested Activities

- Invite students to create patches (made of paper) for a class patchwork quilt in which the colours on their patches show a particular comparison. For example, this patch could be used to illustrate that $\frac{1}{4} < \frac{2}{8}$.



- Have students order a set of unit fractions from least to greatest. Students should be able to defend their order.
- Ask students to estimate the fraction (or decimal) that best represents each 'X'.



- Give students 5 decimal numbers that have friendly fraction equivalents. Keep the numbers between two consecutive whole numbers. For examples 2.5, 2.125, 2.4, 2.75, 3.66. Give them a number line encompassing the same whole numbers and use subdivisions that are only fourths, only thirds or only fifths but without labels. Locate each decimal on the number line and provide the fraction equivalent for each (Van de Walle & Lovin, vol. 3, 2006, p.115).
- Have students place positive decimal numbers and fractions on number lines, such as 2.3 , 2.4, 2.32, 2.36, 2.327.

Possible Models: number lines, fraction pieces, pattern blocks, Cuisenaire rods®, hundredths circle, hundred grids, base ten blocks, place value charts

SCO: **N7: Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:**
• benchmarks
• place value
• equivalent fractions and/or decimals.
[CN, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to estimate a value to make each of the following true:
  a. $0.4 < \frac{?}{8} < 0.7$ b. $\frac{3}{11} < 0.? < \frac{4}{11}$
- Pose questions such as the following:
  a. Which is greater, $\frac{3}{10}$ or $\frac{3}{8}$? Which is greater, $\frac{3}{8}$ or $\frac{7}{10}$? Which is less, $\frac{4}{5}$ or $\frac{3}{4}$?"
  b. Have students explain each selection.
  c. Have students explain how they would compare fractions using their understanding of benchmarks.
- Ask students place the following numbers on a number line that has a few benchmarks labelled:
  $\frac{3}{7}$, $1\frac{1}{3}$, $\frac{5}{9}$, $\frac{13}{12}$, $1\frac{4}{9}$, 0.45, 0.93

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR1: Demonstrate an understanding of oral and written patterns and their equivalent linear relations.** [C, CN, R]
PR2: Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
PR1 Represent and describe patterns and relationships, using graphs and tables. **PR2** Demonstrate an understanding of the relationships within tables of values to solve problems.	**PR1** Demonstrate an understanding of oral and written patterns and their equivalent linear relations. **PR2** Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.	**PR1** Graph and analyze two-variable linear relations.

ELABORATION

Guiding Questions:

• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Mathematics is often referred to as the science of patterns, as they permeate every mathematical concept and are found in everyday contexts. They are usually described as a situation or observed as a pattern, and students have to translate the situation or pattern into an expression or equation.
The various representations of patterns including **physical models**, **table of values**, **algebraic expression**, and **graphs**, provide valuable tools in making generalizations of mathematical relationships. Some characteristics of patterns include the following.

- Patterns include repetitive patterns and growth patterns. Growth patterns are evident in a wide variety of contexts, including arithmetic and geometric situations. Arithmetic patterns are formed by adding or subtracting the same number each time. Geometric patterns are formed by multiplying or dividing by the same number each time. Patterns using concrete/pictorial representations can be written as number patterns, where numbers represent the quantity in each step of the pattern.
- Patterns are used to generalize relationships. How a pattern changes from one term to the next defines the relationship. It explains what you do to the previous number in the pattern to get the next one. An expression that explains what you do to the term to get the value of the pattern for that term is known as a **pattern rule** (Van de Walle & Lovin, vol. 3, 2006, p. 267-268). For example, the number pattern 1, 3, 5, 7, 9,…has the relationship where each number increases by two. The relationship for this pattern is $2n-1$.

Term number (n)	1	2	3	4	5
Term (2n-1)	1	3	5	7	9

Variables such as n are used to represent an unknown quantity. Students should use tables to organize and graph the information that a pattern provides. When using tables, it is important for students to realize that they are looking for the relationship between two variables (**term number and term**). The analysis of graphs should include creating stories that describe the relationship depicted and constructing graphs based on a story which involves changes in related quantities. When students are describing a relationship in a graph they should use language like: as this increases that decreases, as one quantity drops, the other also drops, etc. Students should be able to create a table of values for a given linear relationship and be able to match graphs and sets of linear relationships. From grade six, students should be familiar with the concept of **continuous** and **discrete** data. Continuous data includes an infinite number of values between two points and is shown by joining the data points. Discrete data has a finite number of values (i.e., data that can be counted), and therefore, the points in the graph should not be connected.

SCO: **PR1: Demonstrate an understanding of oral and written patterns and their equivalent linear relations.**
[C, CN, R]
PR2: Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.
[C, CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

PR1
- Formulate a linear relation to represent the relationship in a given oral or written pattern.
- Provide a context for a given linear relation that represents a pattern.
- Represent a pattern in the environment, using a linear relation.

PR2
- Create a table of values for a given linear relation by substituting values for the variable.
- Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).
- Sketch the graph from a table of values created for a given linear relation and describe the patterns found in the graph to draw conclusions; e.g., graph the relationship between n and $2n + 3$.
- Describe the relationship shown on a graph, using everyday language in spoken or written form, to solve problems.
- Match a given set of linear relations to a given set of graphs.
- Match a given set of graphs to a given set of linear relations.

SCO: **PR1: Demonstrate an understanding of oral and written patterns and their equivalent linear relations.**
[C, CN, R]
PR2: Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.
[C, CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

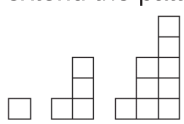
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to describe patterns and rules orally and in writing before using algebraic symbols.
- Provide opportunity to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.
- Provide examples of growth patterns that are arithmetic and geometric.
- Encourage students to draw diagrams and create tables of values to assist them in visualizing the relationship when formulating linear relations representing oral or written patterns.
- Provide experiences representing the same pattern in multiple ways; i.e., model, diagram, table of values, graph, and expression.
- Use real-world contexts as much as possible. Have students represent the pattern by formulating a linear relation.

Suggested Activities

- Have students use cubes to copy and extend the pattern below to the fifth shape in the pattern.



  a. Have students construct a table of values to record and reveal the pattern.
  b. Ask them to explain in words how the pattern grows.
  c. Ask them to predict the total number of cubes needed to make the 10^{th} and 25^{th} shapes in the pattern and explain their prediction.
  d. Ask: If *n* is the number of cubes along the bottom of one shape in the pattern, what would be the total number of cubes in the shape?
  e. Have students make a graph of the pattern and ask what shape the graph has. Discuss the shape of the graph.
- Provide students with the relation 3n - 1 that describes a pattern.
  a. Use this to complete the table of values.
  b. Have students graph the equation, using the table of values.
  c. Ask whether (15, 40) would be in the table of values. Explain why or why not.

n	1	2	3	4	5
3n-1					

Possible Models: linking cubes, colour tiles, toothpicks, counters

SCO: **PR1: Demonstrate an understanding of oral and written patterns and their equivalent linear relations.**
[C, CN, R]
PR2: Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.
[C, CN, R, V]

ASSESSMENT STRATEGIES

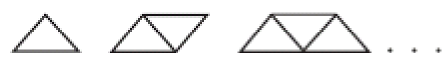
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Show students the diagram below which shows a series of triangular supports for a bridge.
  - Continue the pattern above for up to seven triangles.
  - Complete the chart to show pattern growth.



Term number: (*n*)	1	2	3	4	5	6	7	…	10	…	20
Term (*t*): number of line segments								…		…	

  - Describe in writing how the pattern grows.
  - Write an algebraic expression to show the term (*t*) and for the term number (n).
  - Draw a graph to show the pattern.
  - Does it make sense to join the points? Explain the shape of the graph.
- Provide students with the pattern: 2, 5, 10, 17, 26, 37, ... ,
  - Continue the pattern for the next three numbers.
  - Describe, in words, how the pattern grows. Write the equation.
- Provide students with the following table which shows the relationship between the number of passengers on a tour bus and the total cost of providing boxed lunches.

Passengers (p)	1	2	3	4	5
Cost of lunches (c)	5	10	15	20	25

  a. Ask students to explain how the lunch cost is related to the number of passengers.
  b. Have them write an equation for finding the lunch cost (c) for the number of passengers (p).
  c. Ask them to use the equation to find the cost of lunch if there were 25 people on the tour.
  d. Have students draw a graph to show the relationship in the table of values.
  e. Ask how many people were on the bus if the tour-bus leader spent $200 on box lunches.
- Provide students with a table of values and a graph and ask if they match. Have them justify their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR3: Demonstrate an understanding of preservation of equality by:** **• modeling preservation of equality, concretely, pictorially and symbolically** **• applying preservation of equality to solve equations.** [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
PR5 Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.	**PR3** Demonstrate an understanding of preservation of equality by: • modeling preservation of equality, concretely, pictorially and symbolically • applying preservation of equality to solve equations.	**PR2** Model and solve problems concretely, pictorially and symbolically, using linear equations of the form: • $ax = b$ • $x/a = b$, $a0 \neq$ • $ax + b = c$ • $x/a + b = c$, $a0 \neq$ • $a(x + b) = c$ where a, b and c are integers.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Most students recognize the equal sign as a symbol that tells them to find the answer, but they should understand that the equal sign is a symbol of **equivalence** and balance. While grade seven students have little difficulty with finding the missing number in 7 + 2 = ☐, some may struggle with 3 + 5 = 1 + ☐.

To understand equality, one of the first things students must realize is that equality is a **relationship**, not an operation. Both equality and inequality express relationships between quantities. When the quantities **balance**, there is **equality**. The equal sign is a symbol that indicates the quantity on the left side of the sign is the same as the quantity on the right. When there is an imbalance, there is inequality. The expressions on either side of the equality or **inequality** represent a quantity; e.g., 2 + 3 and $2n + 4$ are both expressions for some quantity. A number sentence is called an **equation**. A number sentence with a **variable** is an **algebraic equation**.

Equality or inequality between quantities can be considered as:

- whole to whole relationships (five red chips = five blue chips or 5 = 5);
- part-part to whole relationships (3 + 5 = 8);
- whole to part-part relationships (8 = 3 + 5);
- part-part to part-part relationships (4 + 4 = 3 + 5).

Solving equations requires that the **balance of the equation** is maintained so that the **expressions** on either side of the equal sign continue to represent the same quantity. For example, if a quantity is added to one side of the equation then, to maintain equality, the same quantity must be added to the other side of the equation. The equality must be maintained similarly for the other operations.

The most useful models for demonstrating preservation of equality are the balance-scale model and algebra tiles. Students must have experiences with concrete models and pictorial representations, before moving to the symbolic. Ensure strong connections are made between each of these representations.

SCO: **PR3: Demonstrate an understanding of preservation of equality by:**
• modeling preservation of equality, concretely, pictorially and symbolically
• applying preservation of equality to solve equations.
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
• What evidence will I look for to know that learning has occurred?
• What should students demonstrate to show their understanding of the mathematical concepts and skills?

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model the preservation of equality for each of the four operations using concrete materials and pictorial representations, explain the process orally and record it symbolically.
- Solve a given problem by applying preservation of equality.

SCO: **PR3: Demonstrate an understanding of preservation of equality by:**
• modeling preservation of equality, concretely, pictorially and symbolically
• applying preservation of equality to solve equations.
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

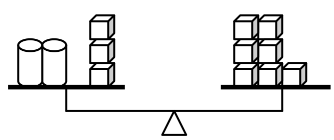
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

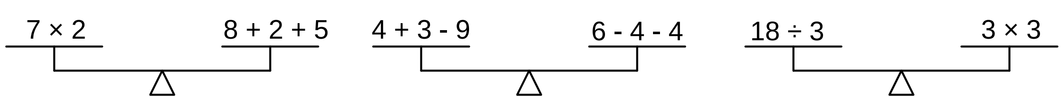
- Build understanding of equality by first using number sentences and exploring what happens when something is changed on one side and what has to be done to compensate for the change in order to preserve equality.
- Have students use balance scales to illustrate an equality and then connect the concrete to the pictorial and symbolic representations.
- Use the pictorial representation of algebra tiles on balance scales to solve equations in which the solution is an integer.
- Have students solve problems by writing the appropriate equation, illustrating the solution concretely or pictorially using balance scales and recording the solution symbolically.

Suggested Activities

- Have students write the equation based on the balance scale model (the pans are balanced and all pieces are positive). Then have them solve the equation both pictorially and symbolically to show the connections between the two.



- Provide students with the balance scales like shown below and ask students:
  a) Are the pans balanced? How do you know?
  b) How can you balance the pans? OR What would happen if you add 4 to the right (or left) side?



- Write two equations that are equivalent to $3n + 1 = 7$ and draw diagrams on the balance scales below to represent the equations (e.g., $3n + 1 - 1 = 7 - 1$; $3n = 6$; $3n + 4 = 10$).

Possible Models: balance scales, algebra tiles, linking cubes, geometric solids

SCO: **PR3: Demonstrate an understanding of preservation of equality by:**
- **modeling preservation of equality, concretely, pictorially and symbolically**
- **applying preservation of equality to solve equations.**

[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

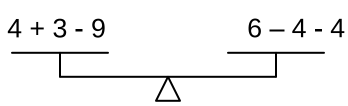
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

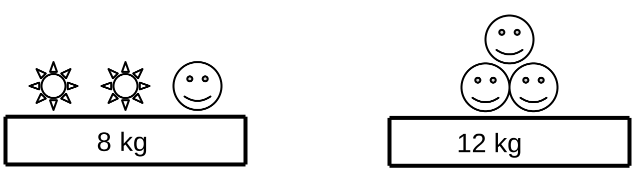
Whole Class/Group/Individual Assessment

- Ask: Will the scales below balance or tilt? How do you know?

- Write two equations that would be equivalent to $2p + 4 = 6$.
  Use a balance scale model to illustrate your equations.
  Consider:



  - Are the pans balanced? How do you know?
  - What would happen if you add 5 to the right hand side?
  - How can you rebalance the pans to preserve the equality?
- Ask: What is the mass of each shape? How do you know?



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR4: Explain the difference between an expression and an equation.**
[C, CN]
PR5: Evaluate an expression given the value of the variable(s).
[CN, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
PR3 Represent generalizations arising from number relationships using equations with letter variables. **PR4** Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.	**PR4** Explain the difference between an expression and an equation. **PR5** Evaluate an expression given the value of the variable(s).	

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

We can use **symbols** to represent a pattern. A **variable** is a symbol that represents an **unknown quantity**. Students are familiar with variables in formulas, such as area = base x height (A = bh). Students might relate variables to things which change over time that are part of their own experiences, such as their height. Some letters used as variables may be confusing to students as they have more than one meaning. For example, "*x*" may be mixed up with the multiplication symbol or "*m"* may be confused with metres. It is important to read expressions such as 3*c* as "a number multiplied by 3" or "3 times *c*". Using contexts may address the confusion of the placement of the variable when writing **expressions** or **equations**. For example, if there are 6 notebooks (*n*) for each student (*s*), they should write $n = 6s$, instead of $s = 6n$.

Previous experience with representing patterns should assist students in understanding the difference between the expressions and equations. An expression is a mathematical phrase composed of numbers and/or variables. An equation is a statement that two expressions are equal. The major difference between an equation and an expression is that an equation expresses equality. For example, $3 + 4 = 7$ is an equation, $3 + y = 7$ is an algebraic equation, and $y + 3$ is an expression. In an expression, the variable can represent any number: $2n + 12$. In an equation, the variable has only one value: $2n = 12$. It is important for students to understand that equations like $x + 6 = 10$ is the same as $10 = x + 6$. Each side of the equation has the same value. A **coefficient** is a quantity (usually a numerical constant), which is multiplied by another quantity following it in an expression or an equation; for example, in the algebraic equation $2p = 10$, the coefficient is 2.

Equations containing more than one variable such as $b = 2n – 1$ can have a variety of values which make them true. For each value of *n*, a corresponding value of *b* can be found. The expression $2n - 1$ behaves in a similar way as the previous equation, whereas $2a - 1 = 7$ is only true for a single value of "*a*".
The equal sign is a symbol that indicates the quantity on the left side of the sign is the same as the quantity on the right. Students should evaluate expressions if given a value. They will also create tables of values when given multiple values for the expression.

SCO: **PR4: Explain the difference between an expression and an equation.**
[C, CN]
PR5: Evaluate an expression given the value of the variable(s).
[CN, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

PR4
- Identify and provide an example of a constant term, a numerical coefficient and a variable in an expression and an equation.
- Explain what a variable is and how it is used in a given expression.
- Provide an example of an expression and an equation, and explain how they are similar and different.

PR5
- Substitute a value for an unknown in a given expression and evaluate the expression.

SCO: **PR4: Explain the difference between an expression and an equation.**
[C, CN]
PR5: Evaluate an expression given the value of the variable(s).
[CN, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Encourage students to describe patterns and rules orally and in writing before using algebraic symbols.
- Provide opportunity to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.
- Introduce the concept of algebraic expressions and equations using real-life or concrete examples. For example, if you are pay a basic fee of $20 a month for a cell phone and are charged $0.90 for each text message, your monthly bill could be determined using the expression $20 + 0.90h$.

Suggested Activities

- Ask students if it can be true that 3b - 1 is equal to 5 under some conditions and equal to 29 in other conditions. Ask students to explain their reasoning.
- Create a set of algebraic expressions and equations on index cards. Create a matching set of cards with the "word' forms. Randomly distribute the cards among the class and have students find the person whose card matches theirs (e.g., $6k + 3$ would match with "three more than six times a number.") The cards could also be used as a "Concentration Game" where cards are turned over two at a time and the player determines if they "match". If the cards do not match, they are turned back over.
- Provide students with some expressions and/or equations in algebraic form, such as below:

  $4p - 5 = b$ $\quad$ $4p - 5$

  $p + 5$ $\quad$ $4p - 5 = 55$

  - List ways in which they are similar and ways in which they differ. Which are equations and which are expressions? Explain why.
  - Have students create a number story for each equation or expression.
- Create a classroom chart with the following headings:

Algebraic Expression	Expression in Words	Variable	Numerical Coefficient	Constant
$3b - 1$	One less than 3 times a number	b	3	-1

  Have students continue to add examples each day to the chart.
- Tell students that Alisha gets $7 per hour to baby-sit. She gets a bonus if she has to baby-sit past 10 p.m. The expression $7h + 3$ represents what Alisha was paid last night. She baby-sat from 5:30 p.m. to 10:30 p.m. What is the variable in this expression? Explain what it represents. What does the coefficient "3" represent in the expression? How much did she earn last night?

Possible Models: two colour counters, algebra tiles, linking cubes

SCO: **PR4: Explain the difference between an expression and an equation.**
[C, CN]
PR5: Evaluate an expression given the value of the variable(s).
[CN, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

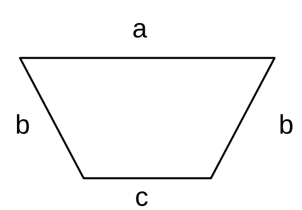
Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Evaluate the following:
  a. $2k + 5$ when $k = \frac{1}{2}$
  b. $5 + 4m$ when $m = 4.2$
  c. $\frac{y}{4} + 22$ when $y = 60$
  d. $-3 + 5q$ when $q = 2$

- Ask students to write an expression that is as simple as possible to represent the perimeter of the following figure:



- Determine which expression has the greater value for each pair of expressions if p = 8.
  a. $p + 7$ $2p$
  b. $10 - p$ $8 \div p$
  c. $3p - 12$ $2 + 2p$
- Ask students to explain expressions and equations in words, similar to the chart in the "Suggested Activities".

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR6: Model and solve problems that can be represented by one-step linear equations of the form *x* + *a* = *b*, concretely, pictorially and symbolically, where *a* and *b* are integers.**
[CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
PR3 Represent generalizations arising from number relationships using equations with letter variables. **PR4** Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.	**PR6** Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where *a* and *b* are integers.	**PR2** Model and solve problems using linear equations of the form: $ax = b$; • $\frac{x}{a} = b$, $a \neq 0$; $ax + b = c$; • $\frac{x}{a} + b = c$ $a \neq 0$; $a(x + b) = c$ concretely, pictorially and symbolically, where *a*, *b*and *c* are integers.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

When solving **one-step equations** in the form of $x + a = b$, *a* and *b* may be integers. However, when solving linear equations that require multiplication or division, only whole numbers should be used as these operations with integers will be addressed in grade eight.

There are many methods for solving a one-step linear equation such as: **inspection**, **systematic trial** (guess and test), **rewriting the equation**, and **creating models** using algebra tiles and using illustrations of balances to show equality. Students should be encouraged to choose the most appropriate method for solving a given problem. Emphasis at this level should be on solving problems **concretely**, **pictorially**, and **symbolically**.

- **Concretely:** Students should be comfortable representing integers with the use of algebra tiles and should continue to do so when modeling an addition or subtraction equation. The zero principle is an important aspect of finding equality between the two sides.
- **Pictorially:** Students need to be encouraged to use concrete models when solving problems and then draw pictures of their models in order to move from the concrete stage to the pictorial.
- **Symbolically:** Students should be made familiar with such notions as adding or subtracting the same value from both sides of an equation and with why equality is maintained.

It is valuable to have students be aware of situations in which they will develop and apply problem solving skills. Students should consider in advance what might be a reasonable solution to the problem, and be aware that once they acquire a solution, it can be checked for accuracy by **substitution** into the original equation.

SCO: **PR6: Model and solve problems that can be represented by one-step linear equations of the form *x* + *a* = *b*, concretely, pictorially and symbolically, where *a* and *b* are integers.**
[CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Represent a given problem with a linear equation and solve the equation using concrete models, e.g., counters, integer tiles.
- Draw a visual representation of the steps required to solve a given linear equation.
- Solve a given problem using a linear equation.
- Verify the solution to a given linear equation using concrete materials and diagrams.
- Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

SCO: **PR6: Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers.**
[CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use a balance scale as a particularly helpful model for illustrating the importance of adding and subtracting like values to each side in order to maintain equality. The *National Library of Virtual Manipulatives* website has an activity to explore the use of balance scales (http://nlvm.usu.edu/en/nav/category_g_3_t_2.html).
- Ask students what other ways an equation can be written. For example, consider how you could rewrite the equation $a + 9 = 14$ (e.g., $14 – 9 = a$). It is through rewriting an equation in alternative forms that students arrive at the notion of solving equations using more formal methods.
- Have students use algebra tiles to represent equations such as: $b + (-4) = 6$. Students should use the tiles to solve the equation and then sketch the tiles they used. This will help them move from solving problems concretely to solving them pictorially.
- Explore the "cover-up" method as an extension. The cover-up method is named for the way it is typically applied. For example, using the formula p + (-5) = 25, cover up the p and ask the question, "What added to (-5) makes 25?"

Suggested Activities

- Using a small envelope, place a number of counters inside. On the outside of the envelope write a variable such as "W". Make an equation such as:

$$\boxed{\text{W}} + 3 = 7$$

  Where, "w" represents the number of counters inside the envelope. Ask students to guess the number of counters in the envelope to make the equation true and then verify by checking the envelope.
- Have students work in pairs to make equations in the form $x + a = b$ (where a and b are integers). Try the following criteria:
  - a is negative.
  - both a and b are negative (or positive).
  - a is positive and b is negative.
- Have students use a calculator to explore what buttons they would use to find the answer for questions such as, $60 \div b = 12$.

Possible Models: algebra tiles, balance scale

SCO: **PR6: Model and solve problems that can be represented by one-step linear equations of the form *x* + *a* = *b*, concretely, pictorially and symbolically, where *a* and *b* are integers.**
[CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to explain how to find the value of *b* in the given equations:
  a. $b + 8 = -13$
  b. $(-6) - b = 81$
  c. $154 + b = 340$
  Ask students to explain if each equation has just one value for b or if they think there are others in addition to the one found. Have students draw a pictorial representation of the steps needed to find the value of *b* and verify their answer by substituting it into the original equation.
- Ask students what integer is hidden in the envelope to make this equation true.

  $\boxed{m} + 2 = 12$

  Ask them how to rewrite the equation so that the two no longer exists on the left-hand side of the equation and yet equality is still maintained. Ask them to use sharing to determine the value of “m”.
- Solve each equation using algebra tiles, and by inspection. Verify each solution by substitution.
  a. $c + 4 = 9$ b. $n - 3 = 8$

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR7: Model and solve problems that can be represented by linear equations of the form:**

- $\boldsymbol{ax + b = c}$
- $\boldsymbol{ax = b}$
- $\boldsymbol{\frac{x}{a} = b, a \neq 0}$

concretely, pictorially and symbolically, where *a*, *b* and *c* are whole numbers.
[CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
	PR7 Model and solve problems that can be represented by linear equations of the form: • $ax + b = c$ • $ax = b$ • $\frac{x}{a} = b$, a ≠ 0 concretely, pictorially and symbolically, where *a*, *b* and *c* are whole numbers.	**PR2** Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b$, a ≠ 0 a. $ax + b = c$ b. $\frac{x}{a} + b = c$, a ≠ 0 • $a(x + b) = c$ concretely, pictorially and symbolically, where *a*, *b* and *c* are integers.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In order for students to solve **linear equations** in the forms $\boldsymbol{ax + b = c; ax = b; \frac{x}{a} = b, a \neq 0}$ they must recognize that the idea of "**balancing**" equations or "moving from one side to another" by using opposite operation(s). This, in fact, allows for the preservation of balance and equality in the equation (left side = right side). In the form $ax + b = c$, students need to perform a **two-step elimination process** to solve for the variable whereas in other equations a single-step process is used. The focus in this outcome is with the use of whole numbers only for a, b and c.

Problem solving is an important skill that we want students to understand and develop. Formulas and equations are seen on a regular basis in our daily lives and it is important for students to understand situations in which they will use, develop and apply such knowledge.

The use of diagrams and concrete materials to demonstrate the idea of solving for "*x*" is a natural progression to lead the students to an understanding of the steps needed to isolate the variable. It is after this progression that students will be able to solve for "*x*" in a linear equation and record the process.

Students should consider in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

SCO: **PR7: Model and solve problems that can be represented by linear equations of the form:**
- ***ax + b = c***
- ***ax = b***
- $\frac{x}{a} = b$, **a ≠ 0**

concretely, pictorially and symbolically, where *a*, *b* and *c* are whole numbers.
[CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model a given problem with a linear equation and solve the equation using concrete models, e.g., counters, algebra tiles.
- Draw a visual representation of the steps used to solve a given linear equation.
- Solve a given problem using a linear equation and record the process.
- Verify the solution to a given linear equation using concrete materials and diagrams.
- Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

SCO: **PR7: Model and solve problems that can be represented by linear equations of the form:**
- ***ax + b = c***
- ***ax = b***
- $\frac{x}{a}$ **= *b*, a ≠ 0**

concretely, pictorially and symbolically, where *a*, *b* and *c* are whole numbers.
[CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies
Consider the following strategies when planning lessons:
- Have students work with materials to model and diagram the idea of balancing and preserving equality with the use of algebra tiles, balances, etc., with the natural progression to formulating and solving written solutions and substituting. After this progression, students will be able to solve for x in a linear equation and record the process: $3x + 2 = 8$

  $3x + 2 - \mathbf{2} = 8 - \mathbf{2}$

  $\frac{3x}{3} = \frac{6}{3}$

  $x = 2$
- Have students consider in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

  Check: $3x + 2 = 8$, where $x = 2$

  $3(2) + 2 = 8$

  $6 + 2 = 8$

  $8 = 8$
- Use the "cover up" method to aid in the initial understanding of the step elimination process. For example, give the equation $4m + 4 = 20$, "cover" the "$4m$" and think, "what amount added to 4 = 20?" Since it is 16 think, "4 × what number = 16?" Recall that 4 × 4 = 16, therefore $m = 4$.

Suggested Activities
- Distribute cards to pairs of students that have one and two-step equations that are shown pictorially, symbolically and concretely. Have students match up the corresponding cards. As an extension, have students create their own cards.
- Provide a step-by-step pictorial representation of a solved equation. Have students provide the symbolic representation of each step. As an extension, all steps except for the answer can be provided and students will have to solve for the coefficient and represent it both symbolically and pictorially.

Possible Models: counters, algebra tiles, linking cubes, balance scale

SCO: **PR7: Model and solve problems that can be represented by linear equations of the form:**
- $\boldsymbol{ax + b = c}$
- $\boldsymbol{ax = b}$
- $\boldsymbol{\frac{x}{a} = b}$**, a ≠ 0**

concretely, pictorially and symbolically, where *a*, *b* and *c* are whole numbers.
[CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

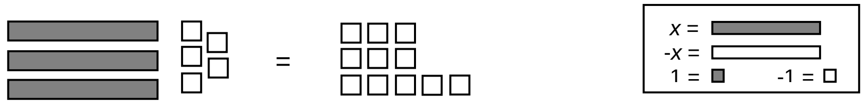
Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to solve the following using algebra tiles and draw pictures to represent the steps taken:
  a. i) $\frac{x}{2} + 1 = 5$
     ii) $2x = 16$
     iii) $4x + 8 = 40$
  b. Ask students what they notice about the answers for each of the equations in part a).
  c. Ask students to analyze the three equations in part a) to determine why the equations had these solutions
- Provide students with a drawing of algebra tiles, such as shown below and ask them to write the equation that is represented. Have students solve the equation and draw and record the steps that were taken.



- Tell students: Susan was given the equation $5j + 7 = 22$ and asked to solve for *j*. She indicated that $j = 15$ but was told that her answer was incorrect. Explain what her misconception was and how you would correct her thinking to correctly solve for *j*.
- Provide students with an equation written in words. For example, four more than twice a number is fifteen.
  a. Write the equation using symbols ($2v + 4 = 15$, or $4 + 2v = 15$).
  b. Use tiles to solve the equation.
  c. Verify the solution by substitution.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS1: Demonstrate an understanding of circles by:**
- **describing the relationships among radius, diameter and circumference of circles**
- **relating circumference to pi**
- **determining the sum of the central angles**
- **constructing circles with a given radius or diameter**
- **solving problems involving the radii, diameters and circumferences of circles.**

[C, CN, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SS1 Demonstrate an understanding of angles by: identifying examples of angles in the environment; classifying angles according to their measure; estimating the measure of angles, using 45°, 90° and 180° as reference angles; determining angle measures in degrees; drawing and labelling angles when the measure is specified. **SS2** Demonstrate that the sum of interior angles is: 180° in a triangle; 360° in a quadrilateral. **SS3** Develop and apply a formula for determining the: perimeter of polygons; area of rectangles; volume of right rectangular prisms.	**SS1** Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference of circles • relating circumference to pi • determining the sum of the central angles • constructing circles with a given radius or diameter • solving problems involving the radii, diameters and circumferences of circles.	**SS3** Determine the surface area of: right rectangular prisms; right triangular prisms; right cylinders to solve problems. **SS4** Develop and apply formulas for determining the volume of right prisms and right cylinders.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

A circle is "a plane (2-D) figure that has all its points the same distance from a fixed point called the **centre** of the circle" (Cathcart 1997, p. 185). The **radius** is the distance from the centre of the circle to the edge of that circle while the **diameter** is a line segment passing through the centre of the circle with both endpoints on the circle. The **circumference** of a circle is the distance around or the perimeter of a circle.

Students should understand that the ratio of circumference to diameter, $\frac{C}{d}$, is constant for all circles, and that the Greek letter **π (pi)** is used to represent the value of this ratio. Pi (π) is an irrational number; it is a non-repeating, non-terminating decimal that cannot be expressed as a fraction (π = 3.1415926535897932384626433832795 ...). The value of π is often approximated as 3.14 although most calculators have a π key. However, for rough estimates, students may use 3 as an approximate value for π.

When measuring the circumference, radius and diameter of circles, the attribute that is being measured is length, and therefore the appropriate units to use include millimetres, centimetres and metres. When finding the sum of the central angles of a circle, the appropriate unit of measure is the degree.

SCO: **SS1: Demonstrate an understanding of circles by:**
- **describing the relationships among radius, diameter and circumference of circles**
- **relating circumference to pi**
- **determining the sum of the central angles**
- **constructing circles with a given radius or diameter**
- **solving problems involving the radii, diameters and circumferences of circles.**

[C, CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Illustrate and explain that the diameter is twice the radius in a given circle.
- Illustrate and explain that the circumference is approximately three times the diameter in a given circle.
- Explain that, for all circles, pi is the ratio of the circumference to the diameter, $\frac{C}{d}$, and its value is approximately 3.14.
- Explain, using an illustration, that the sum of the central angles of a circle is 360°.
- Draw a circle with a given radius or diameter with and without a compass.
- Solve a given contextual problem involving circles.

SCO: **SS1: Demonstrate an understanding of circles by:**
- **describing the relationships among radius, diameter and circumference of circles**
- **relating circumference to pi**
- **determining the sum of the central angles**
- **constructing circles with a given radius or diameter**
- **solving problems involving the radii, diameters and circumferences of circles.**

[C, CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

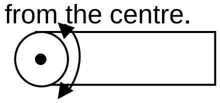
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use exploratory activities to find the value of π.
- Connect the circumference of circles to perimeters of polygons.
- Emphasize that students connect the concrete, pictorial and symbolic representations as they explore the properties of circles.
- Develop the formulas, $C = \pi d$ and $C = 2\pi r$, through exploration activities once the value of π has been established. Students should use these formulas to solve application problems.
- Have students justify the strategies they use in solving problems related to circles and critique strategies used by others.
- Use literature to stimulate students' thinking about circles and their properties, such as *Sir Cumference and the Dragon of Pi* by Cindy Neuschwander.
- Have students construct (draw) circles of various sizes using a variety of construction options (compass, pencil and string, geometry software, etc.). Consider using a non-traditional type of compass. These look like a ruler that rotates around a circle on one end. The centre of the circle is held in place and the pencil is put in one of the holes in the "ruler" (the radius). These are easier for most students to use to construct circles since the pencil is kept a constant distance from the centre.



Suggested Activities

- Investigate the concept of π through measurement and charting the value of $\frac{C}{d}$ for a number of circular objects. Students can bring round containers from home for this purpose. This activity can be done in groups, and results reported to the whole class. A piece of string can be used as the measuring tool. It is useful to perform a similar activity using much larger circles.
- Collect a series of circular containers and ask students to sort them into those for which the circumference is about equal to the height, the circumference is less than the height, and the circumference is more than the height. Ask them to explain their choices and then measure to confirm.
- Have students explore the angles of the various pattern block pieces using the idea that the sum of the interior angles of a circle is 360˚.
- Have students explore the angles of the various tangram pieces using the idea that the sum of the interior angles of a circle is 360˚.
- Have students cut out a quadrilateral, tear off its four angles and place them together. What is the measurement of the four angles together? (360˚) How does this relate to the interior angles of circles?
- Discuss why it is challenging to draw a perfect circle without any tools.

Possible Models: protractor, compass, various circular objects, string, rulers, pattern blocks, tangrams

SCO: **SS1: Demonstrate an understanding of circles by:**
- **describing the relationships among radius, diameter and circumference of circles**
- **relating circumference to pi**
- **determining the sum of the central angles**
- **constructing circles with a given radius or diameter**
- **solving problems involving the radii, diameters and circumferences of circles.**

[C, CN, R, V]

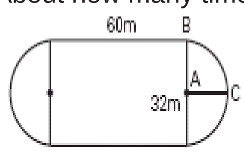
ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

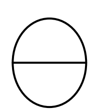
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment
- Ask the student to decide which of the following is the best estimate of the circumference of a circle with a radius 3.5 cm: 10.5 cm, 21cm, or 42 cm. Ask the student to justify his/her choice.
- Tell students: Ali's school has a running track which is semi-circular at each end, as shown. The straight sides are 60 metres and the track is 32 metres wide. About how many times does she have to go around the track to run 2 km?



- Tell students: George's parents are buying a new circular dining-room table. They want the table large enough to seat 12 people so that each person has 60 cm of table space along the circumference.
  a. Ask students what the diameter of the table should be. By how much will the diameter change if George's parents decide to reduce seating space to 45 cm?
  b. Ask students, if each person requires only 45 cm of space around the table, what the smallest possible dimensions of the dining room are, if each chair requires at least 80 cm of space between the table and the closest wall to allow people easy access to their seating place. Ask students what assumptions they made.
- Have students construct circles that meet the following criteria:
  - A circle with a radius of 3 cm.
  - A circle with a diameter of 8 cm.
- Ask students:
  - If you know the radius, what can you do to get the diameter?
  - If you know the diameter, what can you do to get the radius?
- Estimate how many strokes it would take for Kelsey to swim around the edge of the pool, if it took her 30 strokes to swim across the widest part of a circular pool.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*

• *What are the next steps in instruction?*

SCO: **SS2: Develop and apply a formula for determining the area of:** **• triangles** **• parallelograms** **• circles.** [CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SS3 Develop and apply a formula for determining the: perimeter of polygons; area of rectangles; volume of right rectangular prisms.	**SS2** Develop and apply a formula for determining the area of: • parallelograms • triangles • circles.	**SS3** Determine the surface area of: right rectangular prisms; right triangular prisms; right cylinders to solve problems. **SS4** Develop and apply formulas for determining the volume of right prisms and right cylinders.

ELABORATION

Guiding Questions:

• What do I want my students to learn?

• What do I want my students to understand and be able to do?

Area can be defined as a measure of the space inside a region or how many square "units" it takes to cover a region. In using any type of measurement, such as area, it is important to discuss the similarities in developing understanding of the different measures: first identify the **attribute** to be measured, then choose an appropriate unit and finally compare that unit to the object being measured (NCTM, 2000, p. 171). One of the key ideas in understanding area is the attribute of **conservation**: an object retains its size when the orientation is changed or when it is rearranged by subdividing it in any way. Formulas for finding the area of 2-D shapes provide a method of measuring area by using only measures of length (Van de Walle & Lovin, vol. 3, 2006, p. 230). The areas of rectangles, parallelograms, triangles and circles are related, with the area of rectangles forming the foundation for the areas of the other 2-D shapes.

Parallelograms:

Students should recognize that the area of a parallelogram is the same as the area of a related rectangle (one with the same base and height). Students should be able to determine the base or height, given the area and the other dimension, and recognize that a variety of parallelograms can have the same area.

Triangles:

Students should see that the area of a triangle is just one-half of the area of its related parallelogram. They should also be able to connect this idea to the relationship between the formulas of a parallelogram and triangle. Students can use this relationship to find areas of simple triangles. Students should understand that, as long as the base and height are the same, the areas of visually-different triangles are the same.

Circles:

Students should develop the formula for the area of a circle through investigations that connect a circle, cut into equal sectors, to a parallelogram. The exploratory work done by students in estimating the areas of circles, using the square of the radius as a **referent**, also provides a foundation for developing the formula for the area of a circle.

SCO: **SS2: Develop and apply a formula for determining the area of:**
• triangles
• parallelograms
• circles.
[CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain how to estimate the area of a parallelogram.
- Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.
- Generalize a rule to create a formula for determining the area of a parallelogram.
- Apply a formula for determining the area of a given parallelogram.
- Explain how to estimate the area of a triangle.
- Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle.
- Generalize a rule to create a formula for determining the area of a triangle.
- Apply a formula for determining the area of a given triangle.
- Explain how to estimate the area of a circle.
- Illustrate and explain how the area of a parallelogram or a rectangle can be used to determine the area of a circle.
- Generalize a rule to create a formula for determining the area of circles.
- Apply a formula for determining the area of a given circle.
- Solve a given problem involving the area of triangles, parallelograms and/or circles.

SCO: **SS2: Develop and apply a formula for determining the area of:**
- **triangles**
- **parallelograms**
- **circles.**

[CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

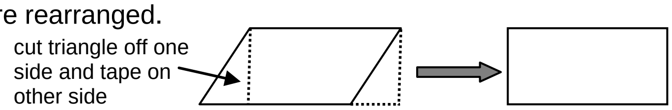
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

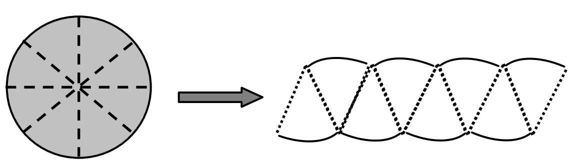
- Have students construct area formulas by applying their knowledge of the area of rectangles and rearranging the shapes of triangles, parallelograms and circles.
- Have students demonstrate conservation of area so that they know the area remains the same when shapes are rearranged.



- Provide students with many types of triangles and parallelograms when constructing meaning for the areas of these 2-D shapes.
- Allow students to recognize the connection between the area of a rectangle and the area of a parallelogram. Then have them construct meaning for the area of a triangle by connecting it to the area of a parallelogram. Finally, have students construct meaning for the area of a circle by rearranging it into a parallelogram or a rectangle. Emphasize the connections among the formulas.
- Ensure students estimate before calculating the areas of parallelograms, triangles and circles.

Suggested Activities

- Make a flexible rectangle using Geo-strips® or cardboard strips and brads. Begin to deform (tilt) the rectangle. Ask the students whether or not the area has changed. Keep tilting until students see that the area has decreased. Discuss how with each additional deformation, a new parallelogram was created with the same base, but less height; therefore, the area decreased.
- Have students cut out a circle, fold it in half, and then fold it in half three more times to develop the formula for the area of a circle. Have students draw a dark line around the circle so the circumference will be apparent. Cut out each sector and line the pieces up to form a "parallelogram" (see below). The radius of the circle represents the height of the parallelogram and the base is represented by half of the circumference = πr. Have them write the formula they have discovered for the area of a circle.



Possible Models: grid paper, Geo-strips®, Power Polygons®, geoboards

SCO: **SS2: Develop and apply a formula for determining the area of:**
- **triangles**
- **parallelograms**
- **circles.**

[CN, PS, R, V]

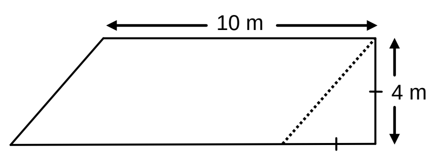
ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

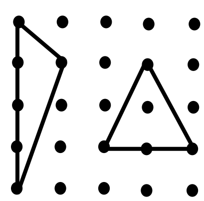
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment
- Ask the student to draw on grid paper a parallelogram with an area of 24 cm^2. Then ask him/her to create three different parallelograms with the same base length and area.
- Tell the students that a triangle and a parallelogram have the same base and the same height. Explain how their areas compare. Include diagrams in your explanation.
- Estimate the area of circular plate that has a radius of 10 cm and explain your thinking. Write the formula for the area of a circle. Calculate the area of the plate. Show all your work.
- Tell the students that a garden plot was made in the shape below (trapezoid). Will the total area of the garden plot be greater than 40 m^2? Explain your thinking. Calculate the total area of the garden plot. Write the formulas that you use. Show all your work.



- Have students create on a geoboard as many different triangles as possible which have an area of 2 cm^2. Students should discover that any triangle with base 4 and height 1, or base 2 and height 2, will have this area.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS3: Perform geometric constructions, including:**
- **perpendicular line segments**
- **parallel line segments**
- **perpendicular bisectors**
- **angle bisectors.**

[CN, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SS4 Construct and compare triangles, including: scalene; isosceles; equilateral; right; obtuse; acute in different orientations.	**SS3** Perform geometric constructions, including: • perpendicular line segments • parallel line segments • perpendicular bisectors • angle bisectors.	**SS5** Draw and interpret top, front and side views of 3- objects composed of right rectangular prisms.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

"What makes shapes alike and different can be determined by an array of geometric properties. For example, shapes have sides that are parallel, perpendicular, or neither." (Van de Walle & Lovin, vol. 3, 2006, p. 179)

Students should be able to identify lines (or **line segments**) that are **parallel** or that are **perpendicular** (meet at right angles) in familiar shapes and in the real world. This might include identifying the parallel sides of squares, rectangles, hexagons, trapezoids, and parallelograms, as well as pairs of adjacent sides that are perpendicular. This should be tied to the properties of the different **polygons**.

Students should come to understand the meaning of bisection through reference to familiar words with the same prefix like bicycle, biplane and bivalve. Another focus for students is the construction of both **bisectors** and **perpendicular-bisectors** of line segments as well as bisectors of angles using a variety of methods. These methods should include paper folding, Miras®, and compass and straightedge. Students should be able to describe how each construction was completed and include the notion of how reflection plays a role with paper folding and Miras®.

Students should be able to explain similarities and differences between line bisectors and perpendicular line bisectors. Reference should also be made to the difference between **intersect** and **bisect**.

SCO: **SS3: Perform geometric constructions, including:**
- **perpendicular line segments**
- **parallel line segments**
- **perpendicular bisectors**
- **angle bisectors.**

[CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors and angle bisectors in the environment.
- Identify line segments on a given diagram that are parallel or perpendicular.
- Draw a line segment perpendicular to another line segment and explain why they are perpendicular.
- Draw a line segment parallel to another line segment and explain why they are parallel.
- Draw the bisector of a given angle using more than one method and verify that the resulting angles are equal.
- Draw the perpendicular bisector of a line segment using more than one method and verify the construction.

SCO: **SS3: Perform geometric constructions, including:**
- **perpendicular line segments**
- **parallel line segments**
- **perpendicular bisectors**
- **angle bisectors.**

[CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students represent parallel or perpendicular lines, or to make a variety of angles, including a right angle, concretely using items like Geo-strips®, toothpicks or straws.
- Teach students to construct perpendicular bisectors of line segments and bisectors of angles using a variety of methods, such as paper folding, use of Miras® and compasses.

Suggested Activities

- Ask students to find a pattern block or Power Polygon® that shows:
  - parallel sides and no right angles;
  - parallel sides and right angles.
- Have students arrange two straws, or two toothpicks, or two Geo-strips® in various configurations (first estimating and then checking):
  - parallel to one another;
  - intersecting;
  - perpendicular at an end point of one straw;
  - perpendicular at endpoints of each straw;
  - one straw perpendicular to the other straw and bisecting;
  - one straw perpendicular to the other straw, but not at its end points and not bisecting;
  - one straw bisecting the other straw but not perpendicular;
  - each straw bisecting the other straw but not perpendicular;
  - one straw bisected by the other straw and perpendicular;
  - each straw bisecting the other straw and perpendicular.
- Have students write the upper case letters of the alphabet that only use line segments. Have them find examples of bisectors of segments, perpendicular segments, and perpendicular bisectors.

Possible Models: Miras®, Geo-strips®, tracing paper, geoboards, dot/grid paper, compass and straight edge, or computer software such as Geometer's Sketchpad®

SCO: **SS3: Perform geometric constructions, including:**
• perpendicular line segments
• parallel line segments
• perpendicular bisectors
• angle bisectors.
[CN, R, V]

ASSESSMENT STRATEGIES

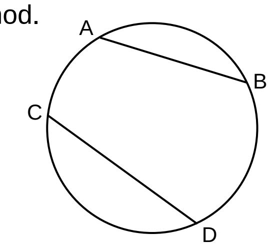
Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

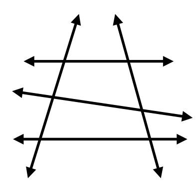
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Draw a line segment approximately 10 cm in length. Construct the perpendicular bisector of this segment, and explain your method.
- Draw an acute angle. Construct the angle bisector. Explain your method.
- Construct the perpendicular- bisectors of line AB and line CD. These bisectors, if done correctly, should meet at the centre of the circle.



- Discuss what the difference is between a line bisector and a perpendicular line bisector.
- Describe a situation when an angle bisector and a line bisector are the same thing.
- Draw a line segment approximately 10 cm in length. Construct a parallel line segment. Explain your method.
- Identify the parallel lines in the following diagram.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS4: Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.** [C, CN, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

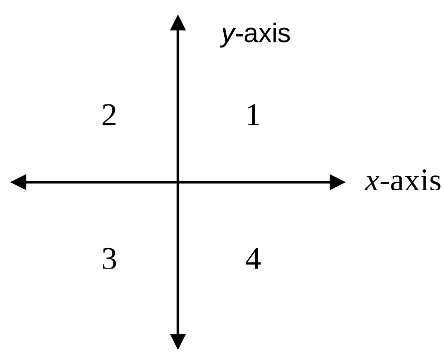
Grade Six	Grade Seven	Grade Eight
SS8 Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.	**SS4** Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.	**PR1** Graph and analyze two variable linear relations.

ELABORATION

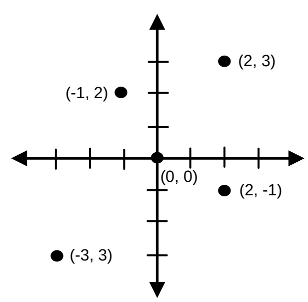
Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should plot data points in all four **quadrants**. **Ordered pairs** of integers can represent position on a four quadrant **Cartesian plane**. The scale of the axes will need to be determined based on the magnitude of the **coordinates**. Students should be exposed to a variety of scales including those with intervals of 1, 2, 5, and 10.



Students should recognize that:
- a negative number for the first coordinate indicates that the point is to the left of the **vertical axis**;
- a negative number for the second coordinate indicates that the point is below the **horizontal axis**;
- the point at which the axes intersect has coordinates (0,0) and is known as the **origin**;
- the position of a point on a grid can be described by its **coordinates;** where the first number is the horizontal coordinate and the second number is the vertical coordinate of the point.



Situations which might be modeled using 4-quadrant graphs include:
- daily high and low temperatures for different days plotted as coordinates;
- mathematical relationships (e.g., a number vs. its double) plotted as coordinates;
- locations, as blocks north, south, east, and west from the town centre plotted as coordinates.

SCO: **SS4: Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.**
[C, CN, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Label the axes of a four quadrant Cartesian plane and identify the origin.
- Identify the location of a given point in any quadrant of a Cartesian plane using an integral ordered pair.
- Plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5 or 10 on its axes.
- Draw shapes and designs, using given integral ordered pairs, in a Cartesian plane.
- Create shapes and designs, and identify the points used to produce the shapes and designs in any quadrant of a Cartesian plane.

SCO: **SS4: Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.**
[C, CN, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

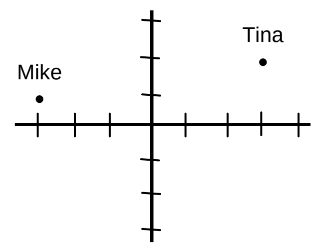
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use four geoboards linked together to represent the four quadrants.
- Provide students with a variety of points that may require them to change the scale on the coordinate grid. For example, (-35, 40) would require students to scale by 5 or 10 instead of a scale factor of 1.
- Create a grid on the floor where students can physically move to identified coordinates.

Suggested Activities

- Show a map (graph) like the one at right. How many blocks north of town centre does Tina live? How many blocks east? Write Mike’s location as an ordered pair.



- Ask students to plot 10 points in quadrant 1 for which the difference between the first and second coordinate is 3. This will create a line. Ask them to find ordered pairs with one or more negative coordinates that are on that line.
- Ask students to plot 10 points for which the first coordinate is the opposite of the second (e.g., (5,-5)). Have them describe the pattern they see and explain why they might have expected that pattern.
- Ask students to plot 10 points on a Cartesian plane. In pairs, students will take turns trying to find each others points similar to the game of Battleship.
- Have students create drawings using all four quadrants of the coordinate grid. They could then provide other students with a list of the vertices, in order, for each drawing created. The other students would subsequently re-create the drawings.
- Use the Internet and research why coordinate planes are often called Cartesian planes. Write a brief paragraph explaining your findings.

Possible Models: geoboards, graph paper, maps

SCO: **SS4: Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.**
[C, CN, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment

- Have students plot the following points on a grid: A (-3, 2), B (1, 2), C (-3, -2). Ask them to determine what the coordinates of a fourth point (D) would be in order to create the square ABCD when the 4 points are connected.
- Have students determine an appropriate grid scale for plotting the following points: (-35, 30), (15, 30), (-20, -20), and (30, -20). Create the grid and plot the points. Ask them to explain why they chose that scale.
- Have students plot points A: (-2, 4) and B: (3, 4). Join the points to create line segment AB. What is the distance between A and B?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS5: Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).** [CN, PS, T, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SS6 Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. **SS7** Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. **SS9** Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).	**SS5** Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).	**SS6** Demonstrate an understanding of tessellation by: explaining the properties of shapes that make tessellating possible; creating tessellations; identifying tessellations in the environment.

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students have been exposed to transformational geometry in previous grades. An emphasis at this level should be the use of the formal language of transformations, such as **translation**, **reflection**, and **rotation**, instead of *slides*, *flips,* and *turns*. Students will be working with transformations and combinations of transformations in all four quadrants of the Cartesian plane.

With respect to describing transformations, students should be able to recognize a given transformation as one of the following: a reflection, a translation, a rotation, or some combination of these. In addition, when given an image and its **translation image**, students should be able to describe:
- a translation, using words and notation describing the translation (e.g., ∆A′B′C′ is the translation image of ∆ABC, or D′(5, 8) is the translation image of D (-5, 8);
- a reflection, by determining the location of the line of reflection;
- a rotation, using degree or fraction-of-turn measures, both clockwise and counter clockwise, and identify the location of the centre of a rotation. A **centre of rotation** may be located on the shape (such as a vertex of the original image) or off the shape.

When investigating properties of transformations, students should consider the concepts of **congruence**, which were developed informally in previous grades. In discussing the properties of transformations, students should consider if the transformation of the image:
- has side lengths and angle measures the same as the given image (congruent);
- is similar to the given image;
- has the same orientation as the given image;
- appears to have remained stationary with respect to the given image.

SCO: **SS5: Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).**
[CN, PS, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

(It is intended that the original shape and its image have vertices with integral coordinates.)

- Identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane.
- Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.
- Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane.
- Determine the distance between points along horizontal and vertical lines in a Cartesian plane.
- Perform a transformation or consecutive transformations on a given 2-D shape and identify coordinates of the vertices of the image.
- Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or a combination of successive transformations.
- Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image.

SCO: **SS5: Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).**
[CN, PS, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

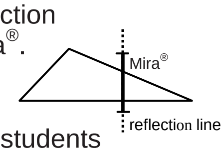
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students use notation such as “the reflection image of A (3, 5) is A’ (-3, 5)” and say “the reflection image of A (3, 5) is A *prime* (-3, 5)”.
- Use grid or dot paper to represent the four-quadrant coordinate plane. Paper folding and the Mira® (transparent mirror) are encouraged in working with reflections. When using paper folding, students can fold on the reflection line and trace the image figure. The Mira® can be placed on the reflection line, as shown, and students can trace the image from the reflection which appears in the Mira®.



- Have students explore rotations where the turn centre of is located off the given image. Many students may still require the use of tracing paper to assist them in placing the transformation of the image.
- Use computer programs, such as Geometer’s Sketchpad®, when available, to assist work with transformations.
- Explore tessellations as a context for applying transformations. It may also be interesting to study art that involves repetitive patterning. These types of activities can often be found on the Internet.

Suggested Activities

- Have students use a geoboard to create an image and a transformation of that image. Have them exchange geoboards with a partner, and have that partner explain the transformation, using specific transformation language, and to describe a transformation that would move the new image back to the pre-transformational position. This process can be repeated using different figures, transformations, and combinations of transformations.
- Reflect a triangle over a line and then reflect it over another line that is parallel to the first. Compare the final image with the pre-image. Describe *one* transformation which would move the transformation back to its original image position.
- Have students work in groups to complete this project. Students select a piece of their favourite music, and choreograph a dance to accompany the music. The dance should include at least two examples of each transformation (translation, reflection, and rotation). Students can perform the dance in class while other students identify the various transformations.

Possible Models: geoboards, geometry sets, Miras®, Geometer’s Sketchpad®, grid paper, coordinate graph paper, tracing paper/wax paper

SCO: **SS5: Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).**
[CN, PS, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment

- Sketch a quadrilateral on a four-quadrant plane. Label and record the coordinates of its vertices.
  a. Translate the quadrilateral by 3 units to the right, and 2 units up.
  b. Label and record the coordinates of the corresponding vertices of the transformation of the image.
  c. Compare the coordinates of the quadrilateral with the coordinates of its transformation image and record your observations.
  d. Predict the coordinates if the quadrilateral is translated 3 units left and 3 units down.
- Ask students to determine what happens to a plotted shape if all the first coordinates are switched with the corresponding second coordinates (e.g., A (3,-2) becomes A′(-2,3)).
- Ask the student to describe where each of these points would be located following a half-turn about the origin: P (-3, -5), Q (3, 6), R (-2, 4).
- Tell students that ∆ABC has the following coordinates: A (1,2), B (3,5) and C (4,0).
  a. Reflect the triangle in the horizontal axis and label the coordinates for ∆A′B′C′.
  b. Reflect ∆A′B′C′ about the vertical axis and label the coordinates for ∆A″B″C″.
  c. Discuss ∆A″B″C″ in relation to the original ∆ABC. Is the transformation of ∆ABC congruent to ∆A″B″C″? Explain. Has the orientation of the transformation of ∆ABC changed? Explain.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP1: Demonstrate an understanding of central tendency and range by:**
- **determining the measures of central tendency (mean, median, mode) and range**
- **determining the most appropriate measures of central tendency to report findings.**

[C, PS, R, T]

SP2: Determine the effect on the mean, median and mode when an outlier is included in a data set.
[C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
	SP1 Demonstrate an understanding of central tendency and range by: • determining the measures of central tendency (mean, median, mode) and range • determining the most appropriate measures of central tendency to report findings. **SP2** Determine the effect on the mean, median and mode when an outlier is included in a data set.	**SP1** Critique ways in which data is presented.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Measures of central tendency allow us to describe a set of data with a single meaningful number. The study of **mean**, **median** or **mode** as measures of central tendency is entirely new to these students in grade seven. The focus of this outcome is to determine mean, median and mode and to understand that situational context will determine which measure is most meaningful. It may be appropriate to use one, two or three of these measures to represent a given data set.

The mean is the sum of the data values divided by the total number of items in the set (arithmetic average). The calculation of the mean describes a set of data by identifying a value obtained from combining all values of the data set and distributing them equally.

The median is the middle number when data are arranged in numerical order. Half of the data values are above the median and half are below. If there are two numbers in the middle of a data set, the median is the mean of those two numbers. The median is not affected by outliers.

The mode is the number that occurs most often in a set of data. It is possible that the data set can have one mode, several modes or no mode at all.

When considering the data as a whole it is often of value to consider the spread or **range** of the data. Students calculate the range by subtracting the smallest data value from the greatest. Range may be used in combination with one of the other measures of central tendency to create a better representation of the data in a set. In a set of data, we often find values which are significantly different from the others. These values are called **outliers**. The presence of outliers may affect which measure of central tendency best represents the data. Students need to explore the effect various outliers have on central tendency.

SCO: **SP1: Demonstrate an understanding of central tendency and range by:**
- **determining the measures of central tendency (mean, median, mode) and range**
- **determining the most appropriate measures of central tendency to report findings.**

[C, PS, R, T]

SP2: Determine the effect on the mean, median and mode when an outlier is included in a data set.
[C, CN, PS, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

SP1
- Determine mean, median and mode for a given set of data, and explain why these values may be the same or different.
- Determine the range of given sets of data.
- Provide a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings.
- Solve a given problem involving the measures of central tendency.

SP2
- Analyze a given set of data to identify any outliers.
- Explain the effect of outliers on the measures of central tendency for a given data set.
- Identify outliers in a given set of data and justify whether or not they are to be included in the reporting of the measures of central tendency.
- Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.

SCO: **SP1: Demonstrate an understanding of central tendency and range by:**
• determining the measures of central tendency (mean, median, mode) and range
• determining the most appropriate measures of central tendency to report findings.
[C, PS, R, T]
SP2: Determine the effect on the mean, median and mode when an outlier is included in a data set.
[C, CN, PS, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use linking cubes to represent a small set of data values to help students explore the mean. For example, with the data set 3, 5, 7 have students build cube towers so it can easily be seen that if they take 2 cubes off the 7 story tower and put it on the 3 story tower that all 3 buildings are now the same size. This will lead to the idea of manipulating the numbers themselves to make them the same. For large data sets a calculator should be used.
- Have students calculate the mean, median and mode for a data set with and without an outlier to see the effect of outliers (change just the lowest or highest value to an outlier). They should see that the median is unaffected but the mean gets either much higher or lower. The mode will usually remain unchanged unless the number changed was the mode.
- Develop a better conceptual understanding of the mean by, building models of the data values with towers of linking cubes and having the students move the cubes so the towers are all the same height.

Suggested Activities

- Have a group of five students (and then six students) line up in increasing order of height to help them understand and visualize the concept of median.
- Have students calculate the mean, median and mode of a set of data from a bar graph.
- Use activities where the outlier is an obvious error to illustrate situations where the outlier would not be used in calculating the averages. If the outlier is not an error it should still be used in calculations, but recognize that the median, in this case, is a better measure of central tendency.
- Create a tri-fold card to define and create examples of each of the measures of central tendency. On each of the outside panels, name and define mean, median or mode. On the corresponding inside panel, create and solve an example of a problem using the measure of central tendency on the front.

Possible Models: linking cubes

SCO: **SP1: Demonstrate an understanding of central tendency and range by:**
- **determining the measures of central tendency (mean, median, mode) and range**
- **determining the most appropriate measures of central tendency to report findings.**

[C, PS, R, T]

SP2: Determine the effect on the mean, median and mode when an outlier is included in a data set.
[C, CN, PS, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment

- Ask students which of mean, median, or mode would be most helpful to know in each situation, and to justify their choice.
  a. You are ordering bowling shoes for a bowling alley.
  b. You want to know if you read more or fewer books per month than most people in your class.
  c. You want to know the “average” amount spent per week on junk food in your class.
- Tell students that the mean of a set of five test scores is 80. One of the grades was erased, but the other four are 90, 95, 85, and 100. What is the missing score?
- Have students write another set of data that would have the same mean and median as 3, 4, 5, 6, 7.
- Ask students for which set of data would it be best to report the median. Explain your answer.
  {5, 7, 11, 19, 28}   {1, 4, 11, 24, 95}

- Create a set of data for each of the following. Each set must have at least six pieces of data.
  a. Situation 1: The mean, median and mode are the same.
  b. Situation 2: The mean, median and mode are different.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP3: Construct, label and interpret circle graphs to solve problems.** [C, CN, PS, R, T, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SP1 Create, label and interpret line graphs to draw conclusions. **SP3** Graph collected data and analyze the graph to solve problems. **SS1** Demonstrate an understanding of angles by: identifying examples of angles in the environment; classifying angles according to their measure; estimating the measure of angles using 45°, 90° and 180° as reference angles; determining angle measures in degrees; drawing and labelling angles when the measure is specified.	**SP3** Construct, label and interpret circle graphs to solve problems.	**SP1** Critique ways in which data is presented.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should realize that a **circle graph** does not show actual measurements. Circle graphs are used to describe how a whole is distributed into its component parts. Data is partitioned into parts and the circle graph illustrates the **ratio** of each part to the whole. The sum of the percent of each part will thus always be a whole or 100%. Likewise, the sum of the central angles will always be 360˚. You may wish to compare and contrast the difference between a circle graph (part to whole) and bar graph (gives actual data value measurements). You can compare two wholes, by comparing two circle graphs. For example, one circle graph may display the percentage of people in each age group for a city and the other may show the same information for the province. Since circle graphs display ratios rather than quantities, the small set of data can be compared to the large set of data. That could not be done with bar graphs (Van de Walle & Lovin, vol.3, 2006, p. 324).

The **title**, **legend** and **labels** are crucial to interpreting circle graphs. Use real data whenever possible when interpreting or drawing circle graphs. When constructing circle graphs, data would typically be given as percents or as raw data that is then converted to percents. Initially students should use hundredths circles to assist them in creating circle graphs; however, they should also learn to construct circle graphs by determining the number of degrees for sections and drawing the graph with a protractor and ruler. Students also need to be able to draw circle graphs using technology.

No matter which approach you take when having students construct graphs, it is important to pose situations that include a real context and have students decide what statistics and what graphs would best serve the purpose (Van de Walle & Lovin, vol.3, 2006, p. 320).

SCO: **SP3: Construct, label and interpret circle graphs to solve problems.**
[C, CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify common attributes of circle graphs, such as:
  title, label or legend;
  the sum of the central angles is 360˚;
  the data is reported as a percent of the total and the sum of the percents is equal to 100%.
- Create and label a circle graph, with and without technology, to display a given set of data.
- Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines and the Internet.
- Translate percentages displayed in a circle graph into quantities to solve a given problem.
- Interpret a given circle graph to answer questions.

SCO: **SP3: Construct, label and interpret circle graphs to solve problems.**
[C, CN, PS, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use data that is real and interesting to students. Newspapers, magazines and Internet sites, such as www.statcan.gc.ca are good resources for data.
- Ensure that the construction of the graph and interpretation of the data are not addressed independently. When students take the time to construct circle graphs, they should be used for interpretation.
- Begin by having students draw circle graphs using a hundredths circle, before students learn the process of converting percentages to degrees.
- Integrate the use of technology to construct graphs after students have experience creating circle graphs with paper-and-pencil methods.

Suggested Activities

- Make a “human circle graph”. Have students choose their favourite of four hockey teams and line them up so that students favouring the same team are together. Have students form a circle. Tape the ends of four long strings in the middle and stretch them out to show the divisions (Van de Walle & Lovin, vol. 3, 2006, p.324).
- Have students make bar graphs. When completed, cut out the bars and tape all of the bars end to end. Tape the two ends together to form a circle. Estimate where the centre of the circle is, draw lines to the points where the different bars meet, and trace around the full loop. Students can now estimate the percentages (Van de Walle & Lovin, vol. 3, 2006, p.324).
- Give students a graph from a text, magazine, or newspaper, and have them convert the graph to some other display form. Discuss which is the better way to display this data and why.
- Using the nutritional information found on food packages, create a circle graph showing the nutritional composition of one serving.

Possible Models: hundredths circle, newspapers, magazines, spreadsheet or graphing computer programs

SCO: **SP3: Construct, label and interpret circle graphs to solve problems.**
[C, CN, PS, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

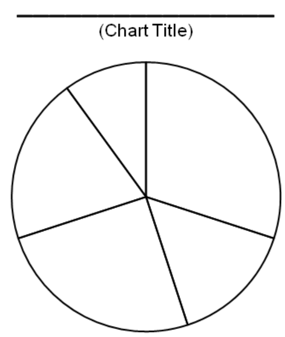
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/One-on-One Assessment

- Ask whether or not the various sections of a particular circle graph can be 35%, 25%, 30%, and 15%? Explain.
- Tell students that Jay works part time in a shoe store. She was involved in completing the spring order. The following were ordered in relation to shoe size:
  5% size 5 - 5 ½
  15% size 6 - 6 ½
  45% size 7 - 7 ½
  20% size 8 - 8 ½
  5% size 9 - 9 ½
  5% size 10 - 10 ½
  a. Construct a circle graph to display this data.
  b. If Jay placed an order for 120 pairs of shoes, how many of each size should she expect to receive?
  c. Write three questions which can be answered from the graph.
- Use the data in the table below; match the correct percentages with the correct sectors in the graph to the right. Label the graph correctly and create an appropriate title for the graph.

Math	30%
Social Studies	15%
Lang. Arts	25%
Science	20%
French	10%



- Provide students with two circle graphs displaying similar data (such as population age distributions from different areas) and have students write comparison statements based on the data shown.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP4: Express probabilities as ratios, fractions and percents.** [C, CN, R, V, T]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SP4 Demonstrate an understanding of probability by: identifying all possible outcomes of a probability experiment; differentiating between experimental and theoretical probability; determining the theoretical probability of outcomes in a probability experiment; determining the experimental probability of outcomes in a probability experiment; comparing experimental results with the theoretical probability for an experiment.	**SP4** Express probabilities as ratios, fractions and percents.	**SP2** Solve problems involving the probability of independent events.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Probability is a measure of how likely an event is to occur. Probability is about predictions of events over the long term rather than predictions of individual, isolated events. **Theoretical probability** can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. For example, in flipping a coin, there are only two possible outcomes, so the probability of flipping a head is, in theory, $\frac{1}{2}$. Often in real-life situations involving probability, it is not possible to determine theoretical probability. We must rely on observation of several trials (experiments) and a good estimate, which can often be made through a data collection process. This is called **experimental probability**.

It is important for students to acquire an understanding that probability can be represented in multiple forms. The probability of an event occurring is most often represented by using a fraction, where the numerator represents the number of favourable outcomes and the denominator represents the total possible outcomes. $P(E) = \frac{\text{\# favourable outcomes}}{\text{\# possible outcomes}}$

This representation has many advantages, since it often maintains the original numbers.
Probability can similarly be represented as a ratio (# favourable outcomes: # possible outcomes).
Likewise, students will often hear in news/weather reports various probability data presented as percents. For example, the likelihood of rainfall for a given day is almost always provided in percent form.
In order for all situations encountered to be meaningful to the student, they should work with a variety of representations of probability.

Students should understand that impossible events have a probability of 0 and events that are certain to occur have a probability of 1.

SCO: **SP4: Express probabilities as ratios, fractions and percents.**
[C, CN, R, V, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction and percent.
- Provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain).

SCO: **SP4: Express probabilities as ratios, fractions and percents.**
[C, CN, R, V, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

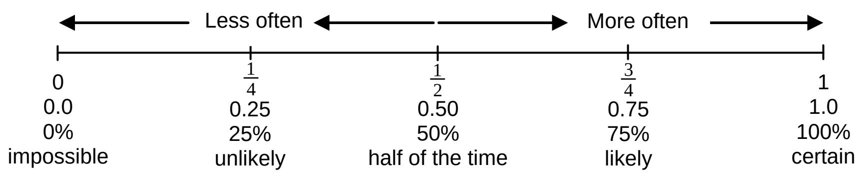
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

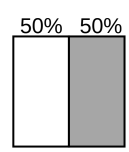
Choosing Instructional Strategies

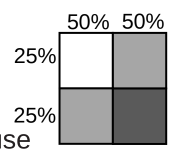
Consider the following strategies when planning lessons:

- Ensure students acquire an understanding that probability can be represented in multiple forms. One means of accomplishing this understanding is by specifying a particular form for the answer.
- Give questions, occasionally, for which no form is specified, or for which different groups are given the same problem but each group is asked to present the answer in a different form. When the class discusses the results in a large group, students should observe the variation in the answers and discuss or account for the differences. Through such experiences, students should come to the realization that the various forms are alternative representations of the same value.
- Review benchmarks and how they relate to probability with students.



Suggested Activities





- Tell students that Lee is a 50% free-throw shooter in basketball. That is, he makes his foul shot 50% of the time. An area model for his first shot would look like the first diagram on the right.
  He also has a 50% chance of making the second shot. He gets 0 points if he makes no baskets, 1 point for one basket, and 2 points for two baskets. The addition of the second shot to the model would look like the second diagram.
  a. Is he more likely to get 0 points, 1 point, or 2 points in a two-shot foul shot situation? Make the diagram on grid paper and use it to determine the probability of getting 0 points, 1 point, or 2 points.
  b. Make a similar area model diagram for Jill, who is a 60% free-throw shooter in basketball, and use it to decide if she is more likely to get 0 points, 1 point, or 2 points. Make the diagram on grid paper and use it to determine the probability of getting 0 points, 1 point, or 2 points.
- Use the information in the table that shows the results of spinning a spinner to find each of the following probabilities. Express your answer as a ratio, as a fraction, and as a percent each time.
  - P (spin of 2)
  - P (spin of 5)
  - P (spin of even number)

2	1	2	4	1
3	2	3	5	2
3	5	1	2	3
1	1	1	4	3
4	2	2	1	1

Possible Models: hundred grid, hundredth circle

SCO: **SP4: Express probabilities as ratios, fractions and percents.**
[C, CN, R, V, T]

ASSESSMENT STRATEGIES

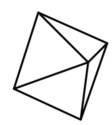
Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students: What does it mean that an event has a probability of 79%? of $\frac{2}{3}$? of 1:5?
- Ask students to provide an example of a situation with a probability of 0.
- Ask students to provide an example of a situation with a probability of 1.
- Tell students a probability of 0 means an event is impossible. A probability of 1 means it is certain. Describe a situation that has a probability of 0.5 occurring. Explain your reasoning.
- Tell students that a bag contains 30 marbles: 7 red, 6 black, 4 yellow, 5 orange and 8 green. What is the probability of drawing a red marble from the bag? Express your answer as a fraction, decimal, and percent.
- Provide students with the following statements: Kari says that the probability that a person's birthday is in the winter is about $\frac{1}{4}$. Andy says it is about 250:1000, and Carson says it is about 25%. Who is right? Explain.
- Describe an event for each of the following probabilities using a single octahedron (8-sided die).
  a. 0 b. 0.25 c. 50% d. $\frac{3}{4}$ e. 5:8 (hint: prime numbers)



- Ask students what was the probability of Sarah rolling a factor of 6 would be if she rolled a six-sided die? Write your answer as a fraction, a ratio, and a percent.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP5: Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**
[C, ME, PS]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SP4 Demonstrate an understanding of probability by: identifying all possible outcomes of a probability experiment; differentiating between experimental and theoretical probability; determining the theoretical probability of outcomes in a probability experiment; determining the experimental probability of outcomes in a probability experiment; comparing experimental results with the theoretical probability for an experiment.	**SP5** Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.	**SP2** Solve problems involving the probability of independent events.

ELABORATION

Guiding Questions:

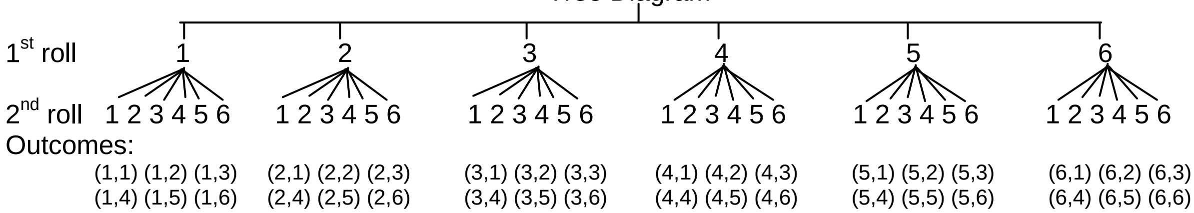
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In grade 7, the study of sample space is limited to **independent events**. Events are considered to be independent if the result of one does not depend on the result of another. The sample space for a probability is the list of all possible outcomes for the events.

- an **outcome** is the result of a single trial of an experiment
- an **event** is one or more outcomes (a set of outcomes) of an experiment.

The sample space of a probability experiment is the set of all possible outcomes for that experiment. The equally likely possible outcomes, or **theoretical probability**, can be represented in a **tree diagram** or **table**. Students will explore several ways to organize the sample space for two independent events. For example, if a number cube was rolled twice, the following tree diagram and table could be created to display the sample space.

Tree Diagram



Table

2nd Roll / 1st Roll	1	2	3	4	5	6
1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

SCO: **SP5: Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**
[C, ME, PS]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide an example of two independent events, such as:
  - spinning a four section spinner and an eight-sided die;
  - tossing a coin and rolling a twelve-sided die;
  - tossing two coins;
  - rolling two dice.

  Explain why they are independent.
- Identify the sample space (all possible outcomes) for each of two independent events using a tree diagram, table or another graphic organizer.

SCO: **SP5: Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**
[C, ME, PS]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

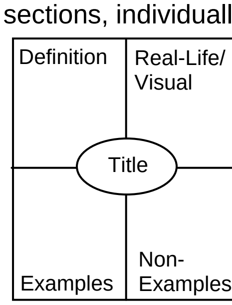
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Clarify terminology used in probability; e.g., sample space, outcomes, events, equally likely outcomes, unequally likely outcomes and trials.
- Provide examples and non-examples of independent events to deepen students' understanding of independent events.
- Provide various strategies for creating the sample space and then calculating the theoretical probability of independent events without using multiplication. These strategies may include using tree diagrams, tables and area models.
- Provide a variety of manipulatives for illustrating independent events; e.g., coins, dice, spinners and drawing cards from a deck or objects from a bag with replacement.

Suggested Activities

- Provide students with various games for two players in which two dice are rolled and rules are given that relate the two numbers rolled. Have students predict whether the games are fair. Encourage students to justify their predictions and then play the games with at least 30 trials to explore.
- Provide students with a template for the Frayer Model and have them fill in the sections, individually or as a group, to consolidate their understanding of independent events.



- Have students predict the probability of getting all questions correct on a test that had 5 multiple-choice questions with four options for each question if they answered them randomly. Students should determine the sample space by creating a tree diagram or a table.
- Tell students that they are helping their little sister pick out an outfit to wear. In her closet, she has a variety of tops and bottoms from which to choose. As tops, she has t-shirts that are blue, green, yellow, red, orange and pink. As bottoms, she has a skirt, a pair of shorts, a pair of capri pants, and a pair of jeans.
  a. What are the two independent events in this example? Explain why these events are independent.
  b. Using an appropriate method, identify the sample space which describes all possible combinations of tops and bottoms you can create for your little sister.
  c. Your mom buys your sister a new purple shirt. How many different outfits can you now create?

Possible Models: number cubes, various spinners, colour tiles or linking cubes, dice with a variety of number of sides, electronic versions of spinners and dice

SCO: **SP5: Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**
[C, ME, PS]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students determine the sample space (all possible outcomes) for the following, using a tree diagram or table.
  a. Jesse has three sweaters and two pairs of shorts. How many different outfits can she create?
  b. A menu offers a lunch special of a hot dog or a hamburger with a choice of an apple, banana, or orange for dessert. How many different combinations of sandwich and dessert could be ordered?
  c. Ling purchased a new cell phone. There is a choice of a hard plastic or leather case and a choice of colours: black, green, blue, or red. How many different combinations of case and colour are possible?
- Tell students that a probability experiment consists of tossing two four-sided fair dice (tetrahedra). Use this information to answer the questions below.

  a. Does this experiment describe two independent events? Explain.
  b. Draw a tree diagram or create a table to show all the possible outcomes for this experiment.
  c. Find the theoretical probability of obtaining a sum of 5 on the two dice in this experiment.
  Show all of your work.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP6: Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.** [C, PS, R, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Six	Grade Seven	Grade Eight
SP4 Demonstrate an understanding of probability by: identifying all possible outcomes of a probability experiment; differentiating between experimental and theoretical probability; determining the theoretical probability of outcomes in a probability experiment; determining the experimental probability of outcomes in a probability experiment; comparing experimental results with the theoretical probability for an experiment.	**SP6** Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.	**SP2** Solve problems involving the probability of independent events.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. It can only be used to predict what will happen in the long run, when events represented are equally likely to occur. Students should realize that the probability in many situations cannot be characterized as equally likely, such as tossing a thumb tack to see if it lands with the point up or down, and therefore theoretical probability is more difficult to determine. In such cases, experiments should be limited to determining the relative frequency of a particular event.

$$\text{Theoretical probability of event } Y = \frac{\text{Number of ways that event } Y \text{ can successfully occur}}{\text{Sample space (total number of possible outcomes)}}$$

Experimental probability or relative frequency of an event is the ratio of the number of observed successful occurrences of the event to the total number of trials. A probability experiment is a method of exploring so students can isolate the critical factors associated with a problem. The greater the number of trials, the closer the experimental probability approaches the theoretical probability. A **single-stage** probability experiment is a probability experiment that involves only one action, such as tossing one coin, to determine an outcome. A **two-stage** probability experiment is a probability experiment that involves two actions, such as tossing a coin and rolling a number cube, to determine an outcome. Two events are independent if the fact that one event occurs does not affect the probability of the second event occurring. If an experiment is conducted by spinning a spinner twice, then a **trial** is the result of spinning the spinner twice and the experimental probability or relative frequency of a specific event.

$$\text{Experimental probability of event } Y = \frac{\text{Number of observed successful occurrences of event } Y}{\text{Sample space (total number of trials in the experiment)}}$$

Before conducting experiments, students should predict the probability whenever possible, and use the experiment to verify or refute the prediction.

SCO: **SP6: Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.**
[C, PS, R, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the theoretical probability of a given outcome involving two independent events.
- Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability to the theoretical probability.
- Solve a given probability problem involving two independent events.

SCO: **SP6: Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.**
[C, PS, R, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

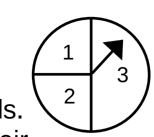
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- In conducting an experiment, the following steps are important:
  - The problem and any underlying assumptions should be clearly defined;
  - A model should be selected to generate the necessary outcomes;
  - A large number of trials should be conducted and recorded;
  - The information should be summarized to draw a conclusion.
- Build on students' understanding of experimental and theoretical probability focusing on a single action from the previous grade and extend it to include two independent events (2 separate actions).
- Ensure students use proper terminology used in probability; e.g., theoretical probability, experimental probability, sample space, outcomes, events, equally likely outcomes, unequally likely outcomes and trials.
- Integrate the use of technology after students have done hands-on work in carrying out experiments with independent events.
- Provide a variety of manipulatives in illustrating independent events; e.g., coins, dice, spinners and drawing cards from a deck or objects from a bag with replacement.
- Have students predict the results of any experiment with independent events by using theoretical probability.

Suggested Activities

- Tell students that an experiment of tossing two fair coins was conducted. About how many times in an experiment with 64 trials would you expect to get two heads? Explain your thinking. Have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain 64 trials and then add more trials as needed to show that experimental probability approaches theoretical probability as the number of trials increases. Have them calculate the experimental probability of getting two heads when two coins are tossed. Have students compare the experimental probability to the theoretical probability. This activity can be extended by having students toss three fair coins and explore how many times they would get two coins with heads.
- Conduct an experiment of spinning the spinner like the one shown at the right twice, and finding the sum of the numbers from the two spins. Predict which sum will appear most often. Explain your thinking. Have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain at least 100 trials. Have students compare the experimental results to their prediction and explain why there may be differences.



- Give students a paper cup and ask them to find the probability it will land on its bottom if dropped. They should see that this is an example of a situation in which they are unable to find the theoretical probability, and so will have to conduct an experiment to find the probability.

Possible Models: number cubes, various spinners, colour tiles, linking cubes, dice with a variety of number of sides, electronic versions of spinners and dice

SCO: **SP6: Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.**
[C, PS, R, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

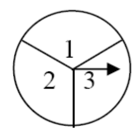
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Tell students that a probability experiment was performed that consisted of tossing a fair coin and spinning a spinner like the one shown below. The outcomes of the probability experiment are shown in the tally chart below.

Outcomes	Tallies
H1	~~IIII~~ ~~IIII~~ ~~IIII~~ II
H2	~~IIII~~ ~~IIII~~ IIII
H3	~~IIII~~ ~~IIII~~ ~~IIII~~ IIII
T1	~~IIII~~ ~~IIII~~ ~~IIII~~
T2	~~IIII~~ ~~IIII~~ ~~IIII~~ III
T3	~~IIII~~ ~~IIII~~ ~~IIII~~ II



  a. How many trials were in the experiment? Explain.
  b. What is the experimental probability of tossing a head and spinning an odd number? Explain.
  c. What is the theoretical probability of tossing a head and spinning an odd number? Explain.
  d. Compare the answers in parts b and c. Explain any discrepancy. What would be the theoretical probability of tossing a head and spinning an odd number if the spinner showed unequally likely outcomes as illustrated to the right? Show all your work.

  This experiment could be modified to include a spinner with more than three sections.
- Tell students that a probability experiment consists of tossing two six-sided fair dice.
  a. Does this experiment describe two independent events? Explain.
  b. Draw a tree diagram or create a table to show all the possible outcomes for this experiment.
  c. Find the theoretical probability of obtaining a sum of 5 on the two dice in this experiment. Show all of your work.
  d. Describe how you could conduct this experiment by using two spinners instead of two six-sided dice.
- Ask students to describe how they could determine the experimental probability for getting 7 out of 10 questions correct on a true-or-false test by guessing alone.

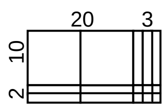
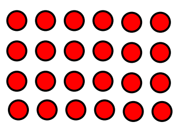
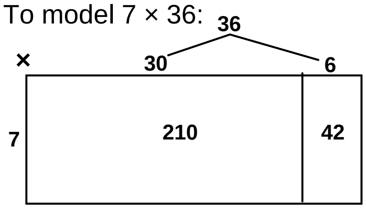
FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

GLOSSARY OF MODELS

This glossary is identical for all grade levels (kindergarten to grade 8). Most of the models have a variety of uses at different grade levels. More information as to which models can be used to develop specific curriculum outcomes is located on the *Instructional Strategies* section of each four-page spread in this curriculum document. The purpose of this glossary is to provide a visual of each model and a brief description of it.

Name	Picture	Description
Algebra tiles		• Sets include "X" tiles (rectangles), "X^2" tiles (large squares), and integer tiles (small squares). • All tiles have a different colour on each side to represent positive and negative. Typically the "X" tiles are green and white and the smaller squares are red and white. • Some sets also include "Y" sets of tiles which are a different colour and size than the "X" tiles.
Area Model	To model 12 × 23: 	• Use base ten blocks to represent the parts of each number that is being multiplied. • To find the answer for the example shown, students can add the various parts of the model: 200 + 30 + 40 + 6 = 276. • This model can also be used for fraction multiplication.
Arrays and Open Arrays	To model 4 × 6:  To model 7 × 36: 	• Use counters arranged in equal rows or columns or a Blackline Master with rows and columns of dots. • Helpful in developing understanding of multiplication facts. • Grids can also be used to model arrays. • Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.
Attribute Blocks		• Sets of blocks that vary in their attributes: o 5 shapes • circle, triangle, square, hexagon, rectangle o 2 thicknesses o 2 sizes o 3 colours
Balance (pan or beam) scales		• Available in a variety of styles and precision. • Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units. • Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.

<table>
<tr><td>Base Ten Blocks</td><td></td><td>

- Include unit cubes, rods, flats, and large cubes.
- Available in a variety of colours and materials (plastic, wood, foam).
- Usually 3-D.

</td></tr>
<tr><td>Beam Balance</td><td colspan="2">☞ see Balance (pan or beam)</td></tr>
<tr><td>Carroll Diagram</td><td>Example:
<table>
<tr><td></td><td>1-digit</td><td>2-digit</td></tr>
<tr><td>Even</td><td>2, 4, 6, 8</td><td>26, 34</td></tr>
<tr><td>Odd</td><td>1, 3, 5, 7, 9</td><td>15, 21</td></tr>
</table>
</td><td>

- Used for classification of different attributes.
- The table shows the four possible combinations for the two attributes.
- Similar to a Venn Diagram.

</td></tr>
<tr><td>Colour Tiles</td><td></td><td>

- Square tiles in 4 colours (red, yellow, green, blue)
- Available in a variety of materials (plastic, wood, foam).

</td></tr>
<tr><td>Counters (two colour)</td><td></td><td>

- Counters have a different colour on each side.
- Available in a variety of colour combinations, but usually are red & white or red & yellow.
- Available in different shapes (circles, squares, bean).

</td></tr>
<tr><td>Cubes (Linking)</td><td></td><td>

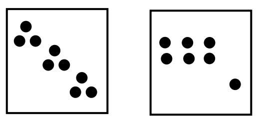
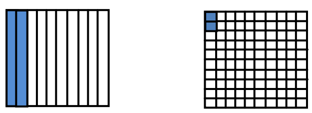
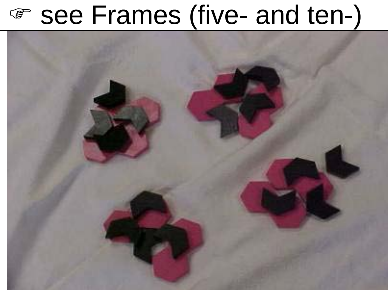
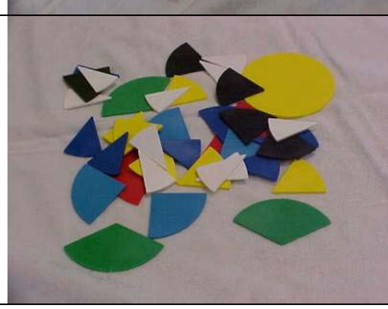
- Set of interlocking 2 cm cubes.
- Most connect on all sides.
- Available in a wide variety of colours (usually 10 colours in each set).
- Brand names include: Multilink, Hex-a-Link, Cube-A-Link.
- Some types only connect on two sides (brand name example: Unifix).

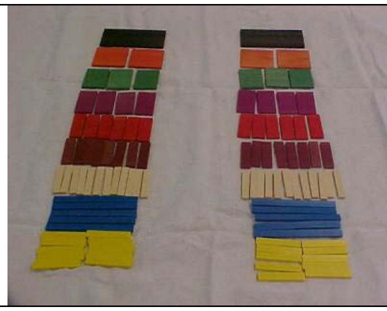
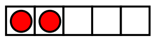
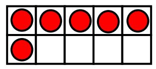
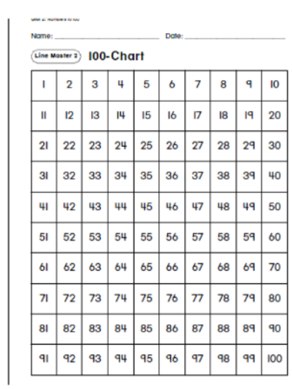
</td></tr>
<tr><td>Cuisenaire Rods®</td><td></td><td>

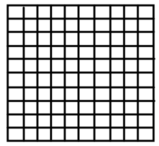
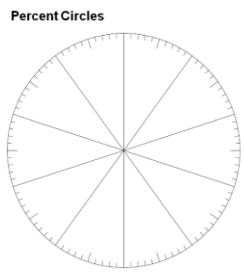
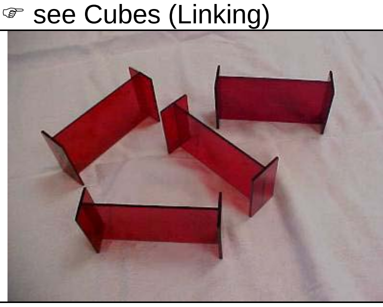
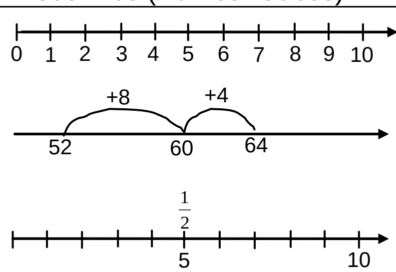
- Set includes 10 different colours of rods.
- Each colour represents a different length and can represent different number values or units of measurement.
- Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange).
- Available in plastic or wood.

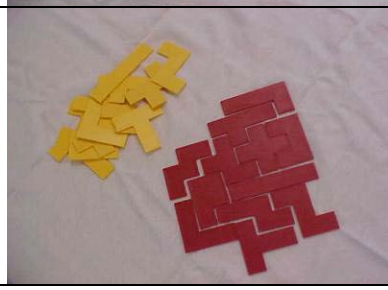
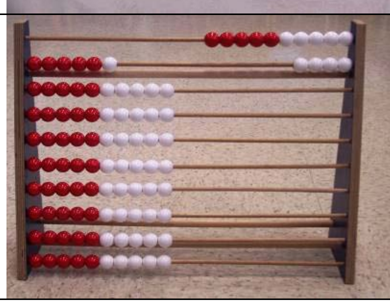
</td></tr>
</table>

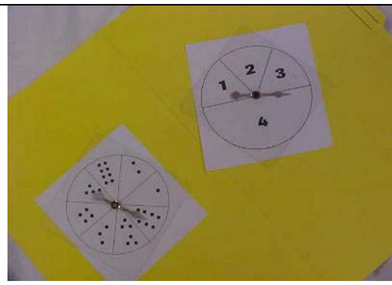
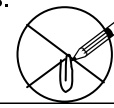
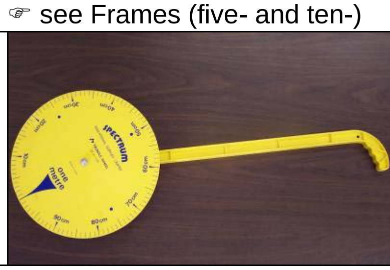
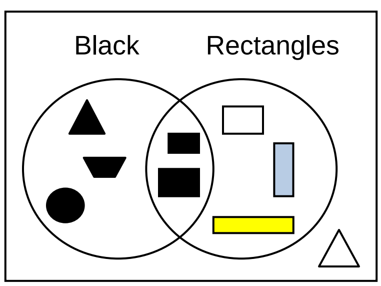
Dice (Number Cubes)		• Standard type is a cube with numbers or dots from 1 to 6 (number cubes).
		• Cubes can have different symbols or words.
		• Also available in:
		  ◦ 4-sided (tetrahedral dice)
		  ◦ 8-sided (octahedral dice)
		  ◦ 10-sided (decahedra dice)
		  ◦ 12-sided, 20-sided, and higher
		  ◦ Place value dice

Dominoes		• Rectangular tiles divided in two-halves. • Each half shows a number of dots: 0 to 6 or • 0 to 9. • Sets include tiles with all the possible number combinations for that set. • Double-six sets include 28 dominoes. • Double-nine sets include 56 dominoes.
Dot Cards		• Sets of cards that display different number of dots (1 to 10) in a variety of arrangements. • Available as free Blackline Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).
Decimal Squares®		• Tenths and hundredths grids that are manufactured with parts of the grids shaded. • Can substitute a Blackline Master and create your own class set.
Double Number Line	☞ see Number lines (standard, open, and double)	
Five-frames	☞ see Frames (five- and ten-)	
Fraction Blocks		• Also known as Fraction Pattern blocks. • 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle. • Use with basic pattern blocks to help study a wider range of denominators and fraction computation.
Fraction Circles		• Sets can include these fraction pieces: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ • Each fraction graduation has its own colour. • It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).

<table>
<tr><td>Fraction Pieces</td><td></td><td>Rectangular pieces that can be used to represent the following fractions: $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$
Offers more flexibility as different pieces can be used to represent 1 whole.
Each fraction graduation has its own colour.
Sets available in different quantities of pieces.</td></tr>
<tr><td>Frames
(five- and ten-)</td><td> </td><td>Available as a Blackline Master in many resources or you can create your own.
Use with any type of counter to fill in the frame as needed.</td></tr>
<tr><td>Geoboards</td><td></td><td>Available in a variety of sizes and styles.
5 × 5 pins
11 × 11 pins
Circular 24 pin
Isometric
Clear plastic models can be used by teachers and students on an overhead.
Some models can be linked to increase the size of the grid.</td></tr>
<tr><td>Geometric Solids</td><td></td><td>Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres.
The number of pieces in a set will vary.
Available in different materials (wood, plastic, foam) and different sizes.</td></tr>
<tr><td>Geo-strips</td><td></td><td>Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes.
Strips come in 5 different lengths. Each length is a different colour.</td></tr>
<tr><td>Hundred Chart</td><td>
Name: Date:
Line Master 3 100-Chart
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100</td><td>10 × 10 grid filled in with numbers 1-100 or 0 - 99.
Available as a Blackline Master in many resources or you can create your own.
Also available as wall charts or “Pocket” charts where cards with the numbers can be inserted or removed.</td></tr>
</table>

Hundred Grid		• 10 × 10 grid. • Available as Blackline Master in many resources.
Hundredths Circle		• Circle divided into tenths and hundredths. • Also known as “percent circles”.
Learning Carpet®	 http://www.thelearningcarpet.ca	• 10 × 10 grid printed on a floor rug that is six feet square. • Number cards and other accessories are available to use with the carpet.
Linking Cubes	☞ see Cubes (Linking)	
Mira®		• Clear red plastic with a bevelled edge that projects reflected image on the other side. • Other brand names include: Reflect-View and Math-Vu™.
Number Cubes	☞ see Dice (Number Cubes)	
Number Lines (standard, open, and double)		• Number lines can begin at 0 or extend in both directions. • Open number lines do not include pre-marked numbers or divisions. Students place these as needed. • Double number lines have numbers written above and below the line to show equivalence.
Open Arrays	☞ see Arrays and Open Arrays	
Open Number Lines	☞ see Number Lines (standard, open, and double)	
Pan Balance	☞ see Balance (pan or beam)	

Pattern Blocks		• Standard set includes: • Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms. • Available in a variety of materials (wood, plastic, foam).
Pentominoes		• Set includes 12 unique polygons. • Each is composed of 5 squares which share at least one side. • Available in 2-D and 3-D in a variety of colours.
Polydrons		• Geometric pieces snap together to build various geometric solids as well as their nets. • Pieces are available in a variety of shapes, colours, and sizes: ▪ Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons • Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.
Power Polygons™		• Set includes the 6 basic pattern block shapes plus 9 related shapes. • Shapes are identified by letter and colour.
Rekenrek		• Counting frame that has 10 beads on each bar: 5 white and 5 red. • Available with different number of bars (1, 2, or 10).

Spinners		• Create your own or use manufactured ones that are available in a wide variety: ◦ number of sections; ◦ colours or numbers; ◦ different size sections; ◦ blank. • Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins. 
Tangrams		• Set of 7 shapes (commonly plastic): ◦ 2 large right-angle triangles ◦ 1 medium right-angle triangle ◦ 2 small right-angle triangles ◦ 1 parallelogram ◦ 1 square • 7-pieces form a square as well as a number of other shapes. • Templates also available to make sets.
Ten-frames	☞ see Frames (five- and ten-)	•
Trundle Wheel		• Tool for measuring longer distances. • Each revolution equals 1 metre usually noted with a click.
Two Colour Counters	☞ see Counters (two colour)	
Venn Diagram		• Used for classification of different attributes. • Can be one, two, or three circles depending on the number of attributes being considered. • Attributes that are common to each group are placed in the interlocking section. • Attributes that don't belong are placed outside of the circle(s), but inside the rectangle. • Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted. • Similar to a Carroll Diagram.

List of Grade 7 Specific Curriculum Outcomes

Number (N)

1. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.
2. Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems.
3. Solve problems involving percents from 1% to 100%.
4. Demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions.
5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).
6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.
7. Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: benchmarks; place value; equivalent fractions and/or decimals.

Patterns & Relations (PR)

(Patterns)

1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations.
2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

(Variables and Equations)

3. Demonstrate an understanding of preservation of equality by: modelling preservation of equality, concretely, pictorially and symbolically; applying preservation of equality to solve equations.
4. Explain the difference between an expression and an equation.
5. Evaluate an expression given the value of the variable(s).
6. Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers.
7. Model and solve problems that can be represented by linear equations.

Shape and Space (SS)

(Measurement)

1. Demonstrate an understanding of circles by: describing the relationships among radius, diameter and circumference of circles; relating circumference to pi; determining the sum of the central angles; constructing circles with a given radius or diameter; solving problems involving the radii, diameters and circumferences of circles.
2. Develop and apply a formula for determining the area of: triangles; parallelograms; circles.

(3-D Objects and 2-D Shapes)

3. Perform geometric constructions, including: perpendicular line segments; parallel line segments; perpendicular bisectors; angle bisectors.

(Transformations)

4. Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.
5. Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).

Statistics and Probability (SP)

(Data Analysis)

1. Demonstrate an understanding of central tendency and range by: determining the measures of central tendency (mean, median, mode) and range; determining the most appropriate measures of central tendency to report findings.
2. Determine the effect on the mean, median & mode when an outlier is included in the data.
3. Construct, label and interpret circle graphs to solve problems.

(Chance and Uncertainty)

4. Express probabilities as ratios, fractions and percents.
5. Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.
6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.

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