Mathematics Grade 5 Curriculum

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New Nouveau Brunswick

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Additional copies of this document may be obtained using the **Title Code: 844470**

Table of Contents

Curriculum Overview for K-9 Mathematics

Background and Rationale.......... 2

Beliefs about Students and Mathematics Learning.......... 2

Goals for Mathematically Literate Students.......... 3
Opportunities for Success.......... 3
Diverse Cultural Perspectives.......... 4
Adapting to the Needs of All Learners.......... 4
Connections Across the Curriculum.......... 4

Assessment.......... 5

Conceptual Framework for K – 9 Mathematics.......... 6

Mathematical Processes.......... 7

Communication.......... 7
Connections.......... 7
Reasoning.......... 7
Mental Mathematics and Estimation.......... 8
Problem Solving.......... 8
Technology.......... 9
Visualization.......... 9

Nature of Mathematics.......... 10

Change.......... 10
Constancy.......... 10
Number Sense.......... 10
Relationships.......... 10
Patterns.......... 11
Spatial Sense.......... 11
Uncertainty.......... 11

Structure of the Mathematics Curriculum.......... 12

Curriculum Document Format.......... 13

Specific Curriculum Outcomes.......... 14

Number.......... 14
Patterns and Relations.......... 54
Shape and Space.......... 62
Statistics and Probability.......... 90

Appendix A: Glossary of Models.......... 102

Appendix B: List of Grade 5 Specific Curriculum Outcomes.......... 109

Appendix C: References.......... 110

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, the Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

GOALS FOR MATHEMATICALLY LITERATE STUDENTS

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

OPPORTUNITIES FOR SUCCESS

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices. Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals. Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

DIVERSE CULTURAL PERSPECTIVES

Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of all students.

Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components. Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. It is important to note that these general instructional strategies may not apply to all students.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

ADAPTING TO THE NEEDS OF ALL LEARNERS

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

CONNECTIONS ACROSS THE CURRICULUM

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

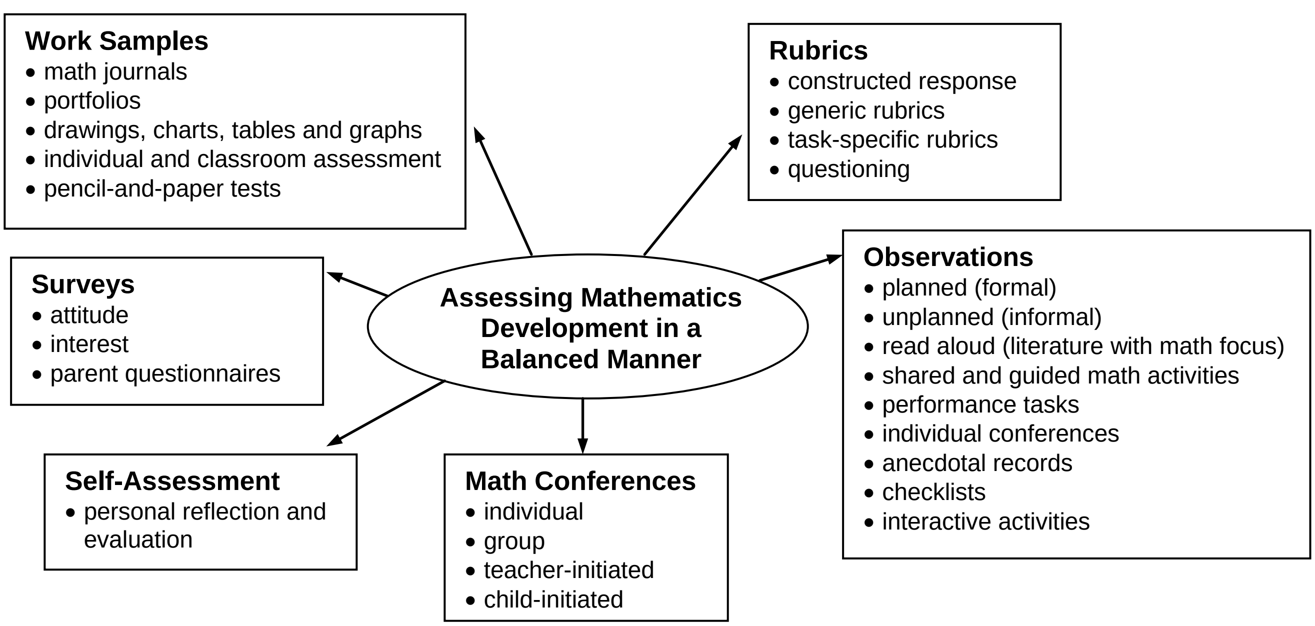
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

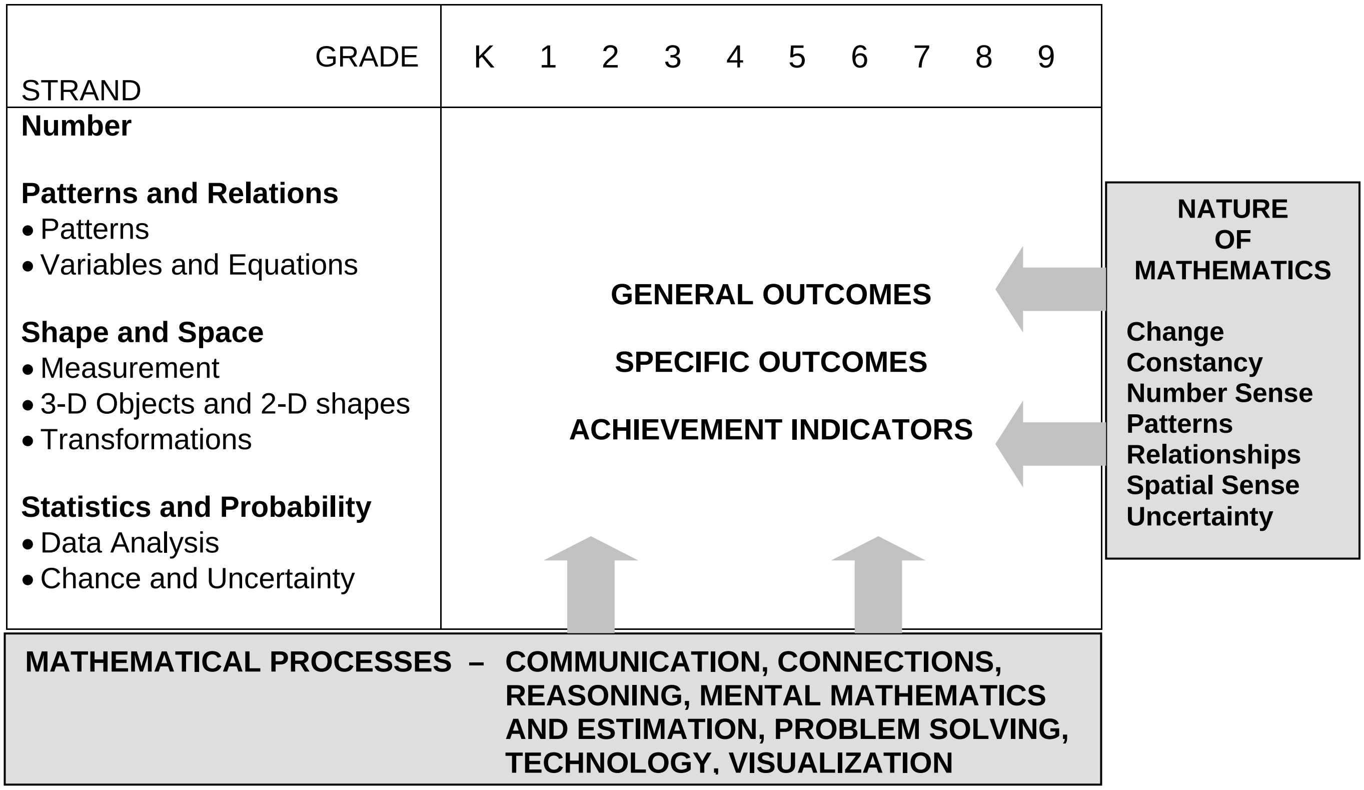
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR K – 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



INSTRUCTIONAL FOCUS

The New Brunswick Curriculum is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.
- There is a greater emphasis on mastery of specific curriculum outcomes.

The mathematics curriculum describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

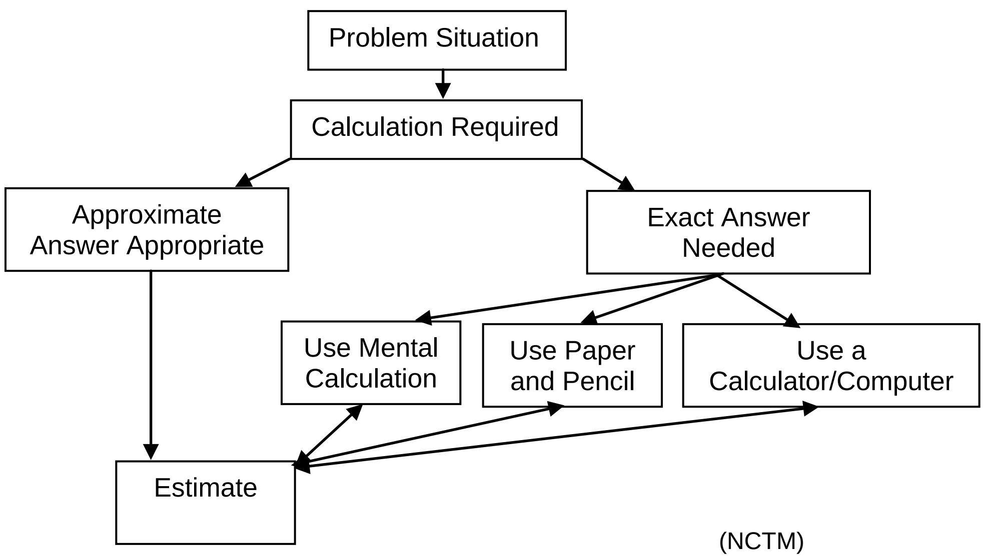
Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mental Mathematics and Estimation [ME]
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005).
Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001). Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below.



(NCTM)

Problem Solving [PS]
Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "*How would you...?"* or "*How could you...?"* the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **relationships**, **patterns**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12 … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

STRUCTURE OF THE MATHEMATICS CURRICULUM

STRANDS

The learning outcomes in the New Brunswick Curriculum are organized into four strands across the grades, K–9. Strands are further subdivided into sub-strands which are the general curriculum outcomes.

OUTCOMES AND ACHIEVEMENT INDICATORS

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement Indicators are one example of a representative list of the depth, breadth and expectations for the outcome. Achievement indicators are pedagogy and context free.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems
	Variables and Equations: Represent algebraic expressions in multiple ways
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them
	Transformations: Describe and analyze position and motion of objects and shapes
Statistics and Probability (SP)	**Data Analysis**: Collect, display and analyze data to solve problems
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The specific curriculum outcomes (SCOs) are presented on individual four-page spreads as illustrated below.

GCO:
SCO: (specific curriculum outcome and mathematical processes)
Key for mathematical processes

Scope and Sequence
Current Grade

Elaboration
Guiding Questions

(Describes the "big ideas" and what the students should learn this year in regards to this concept.)

Page 1

GCO:
SCO:

Achievement Indicators

Guiding Questions

(Describes what could be observed to determine whether students have met the specific outcome.)

Page 2

GCO:
SCO:

Planning for Instruction
Guiding Questions

Choosing Instructional Strategies
(Lists general strategies to assist in teaching this outcome.)

Suggested Activities
(Lists possible specific activities to assist students in learning this concept.)

Possible Models

Page 3

GCO:
SCO:

Assessment Strategies
Guiding Questions

(Overview of assessment)

Whole Class/Group/Individual Assessment
(Lists sample assessment tasks.)

Follow-up on Assessment
Guiding Questions

Page 4

SCO: **N1: Represent and describe whole numbers to 1 000 000.** [C, CN, V, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.	**N1** Represent and describe whole numbers to 1 000 000.	**N1** Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students will continue to use whole numbers as they perform computations or measurements and as they read and interpret data. To have a better understanding of large numbers, such as a million, students need opportunities to investigate problems involving these numbers.
Students should have many opportunities to:

- **read** numbers several ways. For example, 879 346 is read eight hundred seventy-nine thousand, three hundred forty-six but might also be renamed as 87 ten thousands, 9 thousands, 346 ones (other examples may include:8 hundred thousands 79 thousands, 34 tens and 6 ones **or** 879 thousands, 3 hundreds 30 tens and 16 ones). The word "and" is reserved for decimal numbers.
- **record** numbers. For example, ask students to **write** eight hundred thousand sixty; a number which is eighty less than one million; as well as write numbers in **standard form** (741 253) and **expanded notation** (700 000 + 40 000 + 1000 + 200 + 50 + 3). Spaces between groups of three digits are used instead of commas for numbers with more than four digits (e.g., 29 304).
- **establish** personal referents to develop a sense of larger numbers.

It is important for students to recognize and use the conventions for reading and representing numbers in Canada. In Grade 5, students should know that the word "and" is reserved for reading decimal numbers and small spaces are used instead of commas for place value separators.

Through these experiences, students will develop flexibility in identifying and representing numbers up to 1 000 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful. Students should establish **personal referents** to think about large numbers. **Benchmarks** that students may find helpful are multiples of 100, 1000, 10 000 and 100 000, as well as 250 000, 500 000, and 750 000 (quarter, half, and three quarters of a million).
Include situations in which students use a variety of models, such as:

- base ten blocks (e.g., recognize that 1000 large cubes would represent 1 000 000);
- money (e.g., How many $100 bills are there in $9347?);
- place value charts.

Millions			Thousands			Ones		
		O	H	T	O	H	T	O

The focus of instruction should be on ensuring students develop a strong sense of number. The development of this outcome should be ongoing throughout the year.

SCO: **N1: Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Write a given numeral using proper spacing without commas, e.g., 934 567.
- Describe the pattern of adjacent place positions moving from right to left from the ones place.
- Describe the meaning of each digit in a given numeral.
- Provide examples of large numbers used in print or electronic media.
- Express a given numeral in expanded notation, e.g., 45 321 = (4 × 10 000) + (5 × 1000) + (3 × 100) + (2 × 10) + (1 × 1) or 40 000 + 5000 + 300 + 20 + 1.
- Write the numeral represented by a given expanded notation.
- Read a given numeral without using the word “and,” e.g., 574 321 is five hundred seventy-four thousand three hundred twenty-one, NOT five hundred AND seventy-four thousand three hundred AND twenty one. Note: The word “and” is reserved for reading decimal numbers.

SCO: **N1: Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use large numbers from students' experiences, such as populations and professional sport salaries.
- Use visual models based on the cubic centimetre and cubic metre.
- Read and discuss children's books to explore number concepts, such as *How Much Is A Million?* by David Schwartz.
- Provide students with frequent opportunities to read, write, and say numbers in standard and expanded form. Note: insist that students use proper spacing (not commas) when writing large numbers and reserve the use of "and" for reading decimal numbers (no spaces are used for 4-digits).
- Discuss how large numbers can represent either a large amount or a small amount depending on the context used.
- Explore websites, such as Statistics Canada, to explore examples of large numbers.
- Use various manipulatives (number cubes, spinners, number cards, etc.) to generate six-digit numbers. Students can then be asked to explore these numbers in many different ways.

Suggested Activities

- Have students locate large numbers in newspapers or magazines. Ask them to read, write, and represent the numbers in different ways.
- Collect, as a class, some type of object with the objective of reaching a specific quantity. For example, 100 000 buttons, pieces of junk mail or pop can tabs. If collecting is not possible, students could start a project where they draw a specific number of dots each week until the objective is reached.
- Identify how many $100 bills it would take to make $1 000 000.
- Estimate how long a line of 1 million unit cubes would be.
- Ask students questions about the reasonableness of numbers, such as "Have you lived 1 million hours yet?" "Are there 1 million people in any New Brunswick city?" Have students explain their thinking.
- Create 2 page spreads for a class book about 1 million. Each spread could begin: "If you had a million _________, it would be__________." Alternatively the sentences could start, "I wish I had a million _________ , but I would not want a million _________ ."
- Ask students to create six-digit numbers by rolling a number cube six times and order the numbers.
- Have students explore the way numbers have been expressed in examples of whole numbers found in various types of media and personal conversations, and discuss why variations in saying and writing numbers might occur.
- Ask students to compare 10 000 steps to 10 000 metres. If you walked 10 000 steps per day, in how many days would you have walked 1 million steps?
- Ask students to list three non-consecutive numbers between 284 531 and 285 391.
- Have students place counters on a place value chart to represent a number stated orally. The digital form can be written once the chart is filled in, and the number can be read back.

Possible Models: base ten blocks, place value charts, money, number lines, hundred grids, number cards

SCO: **N1: Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to record a series of numbers that have been read to them. Ensure students include correct spacing without commas. Have students express those same numbers in expanded notation.
- Ask, "How does a million compare to 1000, 10 000, 100 000?"
- Have students record a number that is 100 000 more than a given number (or variations of this such as 20 000 less, etc.).
- Ask the students to use newspapers or catalogues to find items that would total $1 million.
- Ask the student how she/he knows that 1 000 000 is the same as 1000 thousands.
- Tell students you bought a car with 50 hundred dollar bills, 20 thousand dollar bills, 100 ten dollar bills and 46 loonies. Ask students to determine the cost of the car.
- Ask a student to describe when 1 000 000 of something might be a big amount? A small amount?
- Provide students with a set of numbers (up to seven-digits) written in words and have students write the numbers in standard form using correct spacing and no commas.
- Place two zeros anywhere in the number 3759 to form a new six-digit number. Read the new number and explain how the value of each digit has changed.
- Provide students with a set of numbers written in expanded form and have students write them in standard form. For example:
  - $(2 \times 100\,000) + (5 \times 1000) + (6 \times 100) + 9$
- Provide students with a set of numbers written in standard form and have students write them in expanded notation. For example:
  - 40 109
- Explain how the value of the "1" digit changed in each of the following numbers:
  - 2681
  - 1 000 000
  - 918 702
  - 103 557

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N2: Use estimation strategies including:** • **front-end rounding** • **compensation** • **compatible numbers** **in problem-solving contexts.** [C, CN, ME, PS, R, V]
[C] Communication **[T]** Technology **[PS]** Problem Solving **[V]** Visualization **[CN]** Connections **[R]** Reasoning **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: using personal strategies for adding and subtracting; estimating sums and differences; solving problems involving addition and subtraction.	**N2** Use estimation strategies including: • front-end rounding • compensation • compatible numbers in problem-solving contexts.	

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Students need to recognize that estimation is a useful skill in their lives. To be efficient when estimating sums, differences, products, and quotients mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Students should be aware that in real-life estimation contexts **overestimating** is often important.

The context and the numbers and operations involved affect the estimation strategy chosen.

- **Front-end Rounding**: There are a number of things to consider when rounding to estimate for a multiplication calculation. If one of the factors is a single digit, consider the other factor carefully. For example, when estimating 8 x 693, rounding 693 to 700 and multiplying by 8 is a much closer estimate than multiplying 10 by 700. Explore rounding one factor up and the other one down, even if it does not follow the "rounding rule" of going to the next closest multiple of 10 or 100. For example, when estimating 77 by 35, compare 80 x 30 and 80 x 40 to the actual answer of 2695.
- **Compensation**: In this case, compensation refers to increasing one value and decreasing the other. For example, 35 + 57 might be estimated as 30 + 60 (rather than 40 + 60) as this is a more accurate estimation.
- **Compatible numbers** or “nice numbers”: Clustering compatible (or near compatible) numbers is useful for addition. For example, to solve 134 + 55 + 68 + 46, the 46 and 55 together make about 100; the 134 and 68 make about another 200 for a total of 300. Look for compatible numbers when rounding for a division estimate. For 477 ÷ 6, think "480 ÷ 6". For 332 ÷ 78, think "320 ÷ 80”.

Students and teachers should note that multiplication and division estimations are typically further from the actual value because of the nature of the operations involved.

SCO: **N2: Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide a context for when estimation is used to:
  - make predictions
  - check reasonableness of an answer
  - determine approximate answers.
- Describe contexts in which overestimating is important.
- Determine the approximate solution to a given problem not requiring an exact answer.
- Estimate a sum or product using compatible numbers.
- Estimate the solution to a given problem using compensation and explain the reason for compensation.
- Select and use an estimation strategy for a given problem.
- Apply front-end rounding to estimate:
  - sums, e.g., 248 + 627 is more than 200 + 600 = 800
  - differences, e.g., 974 – 250 is close to 900 – 200 = 700
  - products, e.g., the product of 23 × 24 is greater than 20 × 20 (400) and less than 25 × 25 (625)
  - quotients, e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 (200).

SCO: **N2: Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

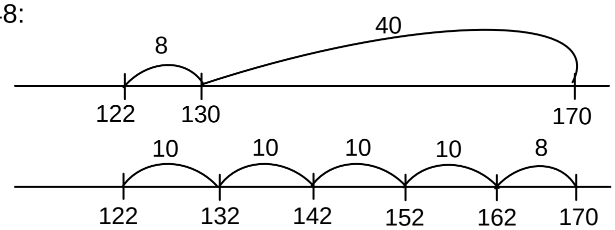
PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies
Consider the following strategies when planning lessons:
- Support students in exploring personal strategies for estimation, but then guide students toward more efficient and accurate strategies as needed. Ask: Is your estimation accurate? Why?
- Have students share personal strategies. Begin with the least efficient strategies and then share progressively more complex as this encourages participation from all and does not discourage others.
- Accept a range of estimates, but focus on “best” estimates.
- Provide real-world contexts for estimations, as most situations require estimations and not precise answers.
- Practice strategy selection and explain the choice for estimation.

Suggested Activities
- Tell the student that □83 + 190 is about 600. What digit should go in the box?
- Ask the students to find two numbers with a difference of about 150 and a sum of about 500 or two numbers with a difference of about 80 and a sum of about 200.
- Ask the student to estimate what one might subtract in each case below so that the answer is close to, but not exactly, 50:
  - 384 - ____
  - 219 - ____
  - 68 - _____
- Have students describe a real world situation when overestimating is appropriate.
- Ask students if they have lived closer to 400, 4000, or 40 000 days. Explain their thinking.
- Have students use an open number line to help visualize the numbers and model their strategies. For example to solve 170 – 48:



Possible Models: number lines (including open number lines), calculator

SCO: **N2: Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask: Which pair of factors would you choose to estimate 37 x 94? Explain why.
  30 x 90 40 x 100 35 x 95 40 x 95 40 x 90
- Have students estimate each sum and explain their strategies: 1976 + 3456 69 423 + 21 097
- Have students estimate each difference and explain their strategies: 99 764 – 17 368 5703 – 755
- Have students add 6785 + 1834. Explain how they know their answer is reasonable using estimates in their explanation.
- Have students solve problems that require an estimate, such as: Jeff has 138 cans of soup. He wants to collect 500 cans for the food bank. About how many more does he need to collect?
- Ask the student for an estimate if a number between 300 and 400 is divided by a number between 60 and 70.
- Tell students that a bus holds 58 students. How would you estimate how many buses are needed to transport 3000 students?
- Tell the student that you have multiplied a 3-digit number by a 1-digit number and the answer is about 1000. Ask the student to write three possible pairs of factors.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N3: Apply mental mathematics strategies and number properties, such as:** • **skip counting from a known fact** • **using doubling or halving** • **using patterns in the 9s facts** • **using repeated doubling or halving** **to determine answers for basic multiplication facts to 81 and related division facts.** [C, CN, ME, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Math and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N5 Describe and apply mental mathematics strategies, such as: skip counting from a known fact; using doubling or halving; using doubling or halving and adding or subtracting one more group; using patterns in the 9s facts; using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.	**N3** Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.	**N3** Demonstrate an understanding of factors and multiples by: determining multiples and factors of numbers less than 100; identifying prime and composite numbers; solving problems involving multiples.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

This is an extension of the Grade 4 outcomes, N4 and N5. The goal for Grade 5 is **automaticity**, which means that students are able to recall multiplication facts with little or no effort. The fact recall should be automatic as a result of thinking about the relationship between the facts and extensive use of strategies. Students need to understand and use the relationship between multiplication and division. Students should recognize that multiplication can be used to solve division situations. Providing students with contextual problems to solve is a critical part of this process.

In Grade 4, students will have become proficient at **doubling** (e.g., 4 × 3 = (2 × 3) × 2). This idea is extended in Grade 5 to include **repeated doubling**. For example, to solve 8 x 6, students can think 2 × 6 = 12 and 4 × 6 = 24, so 8 × 6 = 48. The same principle applies to **halving** and **repeated halving**. For example, for 36 ÷ 4, think 36 ÷ 2 = 18; so 18 ÷ 2 = 9.

Skip counting up or down from a known fact reinforces the meanings of multiplication and division as students must be thinking about the addition or subtraction of "groups". For example, for 8 × 7, think 7 × 7 = 49 and then add another group of 7; 49 + 7 = 56.

Students should be given opportunities to explore and discover the many patterns that exist in the table of nines facts. Have students use the patterns they find to create strategies to determine a nines fact that is not known.

Along with understanding why multiplication by 0 produces a product of 0, students must be able to explain why **division by 0 is undefined** or **not possible**. It is not possible to make a set of zero from a given group, nor is it possible to make zero sets from a given group. When demonstrated as repeated subtraction, removing groups of zero will never change your dividend. Rather than telling students these properties, pose problems involving 0.

SCO: **N3: Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Describe the mental mathematics strategy used to determine a given basic fact, such as:
  - skip count up by one or two groups from a known fact, e.g., if 5 × 7 = 35, then 6 × 7 is equal to 35 + 7 and 7 × 7 is equal to 35 + 7 + 7
  - skip count down by one or two groups from a known fact, e.g., if 8 × 8 = 64, then 7 × 8 is equal to 64 – 8 and 6 × 8 is equal to 64 – 8 – 8
  - doubling, e.g., for 8 × 3 think 4 × 3 = 12, and 8 × 3 = 12 + 12
  - patterns when multiplying by 9, e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63
  - repeated doubling, e.g., if 2 × 6 is equal to 12, then 4 × 6 is equal to 24 and 8 × 6 is equal to 48
  - repeated halving, e.g., for 60 ÷ 4, think 60 ÷ 2 = 30 and 30 ÷ 2 = 15.
- Explain why multiplying by zero produces a product of zero.
- Explain why division by zero is not possible or undefined, e.g., 8 ÷ 0.
- Recall multiplication facts to 81 and related division facts.

SCO: **N3: Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:
- Use a problem solving context to invent strategies and practice facts.
- Introduce and practice strategies. When students are proficient at more than one strategy, have them explain why one strategy may be better than another in a given situation.
- Have students start with facts that they know. Allow students to use counters, base ten blocks, colour tiles and arrays as they continue to develop strategies.
- Ensure students understand why strategies work. Fact strategies should not become “rules without reasons” (Van de Walle & Lovin, vol. 2, 2006; p. 90).
- Provide students with situations that involve division by zero. For example, if you have 8 counters, how many sets of zero can be made or how many times can you subtract 0 from 8 to get to 0 ($8 \div 0$)? This can also be explored using the relationship between multiplication and division. To solve $8 \div 0$, students could try to use multiplication, but discover that there is no answer for $0 \times \square = 8$.
- Play games to practice strategies that lead to fact recall.
- Avoid using drill until students have mastered a strategy. Unless students have mastered a strategy, drills are not effective.

Suggested Activities

- Use counters to model 6×6 in an array. Add another row or column to demonstrate a related fact.
- Have students explore and share strategies to determine answers to unknown facts.
- Ask students, if Jennifer reads a chapter of a novel each day, how many chapters will she have read in 8 weeks? Describe your strategy.
- Have students agree or disagree with this statement: “There are more than 2 ways to figure out any multiplication fact”. Have students use a fact of their choice.
- Ask students if they agree or disagree with this statement: “If you know your multiplication facts, you already know your division facts”. Have students provide a rationale.
- Provide small groups with a square piece of paper. Have them fold the paper in half and record how many sections they have. Have them fold it again and record how many sections. Have them continue until they see a pattern of doubling. Relate this to halving.
- Have students fill in all the facts they know in a multiplication table. Ask them to work with a partner to identify what strategies they could use to fill in the rest of the table.
- Use sets of “loop cards” (I have ___, who has ____) where the answer for one card answers the question on another to form a loop of questions and answers. For example, one card could read, “I have 24. Who has 3×4?”

Possible Models: counters, colour tiles, base ten blocks, number lines, array models, area models

SCO: **N3: Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment
- Ask students to list three multiplication facts they can use to help calculate 5 x 8, and explain how they can use each fact.
- Ask, "If you buy muffins in boxes of 6, how many muffins are in 7 boxes? How would the number of muffins change if you bought 9 boxes? If you needed 36 muffins for a party, how many boxes would you buy?"
- Tell students that Scott solved the problem $48 \div 8$ by thinking: $48 \div 2 = 24$, then $24 \div 2 = 12$ and finally $12 \div 2 = 6$. Explain the strategy Scott used.
- Ask students how they could use multiplication to find the perimeter of a square.
- Have students use manipulatives to explain why $7 \times 0 = 0$ and $0 \times 9 = 0$.
- Have students use manipulatives to explain why $6 \div 0$ is not possible.
- Tell the students that you have eight boxes, each of which holds six markers, and one other box that has only five markers in it. Ask the students to describe at least two ways one could find the total number of markers, and to explain which strategy they would prefer and why.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N4: Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N5 Describe and apply mental mathematics strategies, such as: skip counting from a known fact; using doubling or halving; using doubling or halving and adding or subtracting one more group; using patterns in the 9s facts; using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.	**N4** Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property.	

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

A **mental computation** enables students to determine answers without paper and pencil and enhances flexible thinking. In Grade 5, students are extending the strategies learned in Grade 4 to multiply mentally. It is important to recognize that these strategies develop and improve over the years with regular practice. This means that mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades. Mental strategies must be taught both explicitly as well as being embedded in problem solving situations. Sharing of computational strategies within the context of problem-solving situations is essential.

Students should perform and discuss the following types of mental multiplication on a regular basis:
- **Annexing then adding zero**: For multiplication by 10, 100 and 1000 and multiplication of single-digit multiples of powers of ten (e.g., for 30 × 400, students should think "Tens times hundreds is thousands. How many thousands? 3 × 4 or 12 thousands."). Multiplying by powers of ten does not change the digits of a number, only the position of each digit within the number (Small, 2008, p. 238). Students need to explore why this strategy works using materials and smaller numbers so they are able to understand the place value patterns that happen when multiplying by powers of ten.
- **Halving and doubling**: For example, to solve 4 × 16, students can change it to 2 × 32 or 8 × 8.
- **Distributive property**: The ability to break numbers apart is important in multiplication. For example, to multiply 5 × 43, think 5 × 40 (200) and 5 × 3 (15) and then add the results. This principle also applies to multiplication questions in which one of the factors ends in a nine (or eight or seven). For such questions, one could use a **compensating strategy** - multiply by the next multiple of ten and compensate by subtracting to find the actual product. For example, when multiplying 39 by 7 mentally, one could think, "7 times 40 is 280, but there were only 39 sevens so I need to subtract 7 from 280 which gives an answer of 273."

Whenever presented with problems that require computations, students should be encouraged to first check to see if it can be done mentally. Students should select an efficient strategy that makes sense to them and consistently produces accurate results.

SCO: **N4: Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the products when one factor is a multiple of 10, 100 or 1000 by annexing zero or tacking on zeros, e.g., for 3 × 200 think 3 × 2 hundreds which equals six hundreds (600).
- Apply halving and doubling when determining a given product, e.g., 32 × 5 is the same as 16 × 10; 18 × 15 is the same as 9 × 30; 48 × 25 is the same as 24 × 50 which is the same as 12 × 100.
- Apply the distributive property to determine a given product involving multiplying factors that are close to multiples of 10, e.g., 98 × 7 = (100 × 7) – (2 × 7).

SCO: **N4: Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many experiences to construct a personal strategy and then be guided to use the most efficient strategy available. Mental strategies encourage students to think about the whole number and not just the digits.
- Provide students with frequent opportunities to share their mental strategies.
- Provide problem-based situations that support the use of mental strategies.
- Use materials and pictorial representations to demonstrate mental strategies.
- Introduce a strategy with the use of materials, practice the strategy, and continue to introduce and practice new strategies. When students have two or more strategies, it is important to encourage them to choose the most efficient strategy for the student.
- Encourage students to visualize the process for the strategy they are using.
- Place students in pairs to practice strategies as well as strategy selection.
- Avoid timed tests until students have developed and practiced specific mental strategies in other contexts.
- Ask students to keep track of when they use their mental math strategies outside of the classroom and to write about these experiences.
- Ask the students to keep a list of mental math strategies that they use regularly.

Suggested Activities

- Use two recipe cards and have students write a series of mental math questions. Students take the cards home to have a “race” with a parent/guardian. The student can then “teach” the strategy being practiced at home.
- Ask students to explain how they could calculate 23 × 8 if the “eight” key on the calculator was broken.
- Prepare cards with number sentences that can be solved using two or more strategies. Put these into a single package. Prepare simple pictures or labels for the strategies in the package. Have students sort the problems and then solve them using an appropriate strategy.
- Ask the student to use square tiles to show that if the length of a rectangle is halved and the width is doubled, the area remains the same.
- Ask the student to provide an explanation and examples for how to multiply a 1-digit number by 99 mentally.

Possible Models: base ten blocks, counters, place value charts, array model, area model

SCO: **N4: Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Tell the student that when asked to multiply 36 × 11, Kelly said, "I think 360 + 36 = 396." Ask the student to explain Kelly's thinking.
- Ask: Why is it easy to calculate the questions below mentally?
  48 × 20 50 × 86
- Ask the student why Scott multiplied 11 × 30 to find 22 × 15.
- Provide students with a problem situations to solve, such as:
  - Fourteen students raised $20 each in pledges for “Save the Wetlands Walk”. How much money was raised? How much money would be raised if the pledges were increased to $50 each?
  - A hotel has 7 floors with 39 windows on each floor. How many windows are in the hotel? Explain how you know.
- Explain how you know that 48 × 50 is the same as 24 × 100.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.** [C, CN, PS, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: using personal strategies for multiplication with and without concrete materials; using arrays to represent multiplication; connecting concrete representations to symbolic representations; estimating products.	**N5** Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.	**N8** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

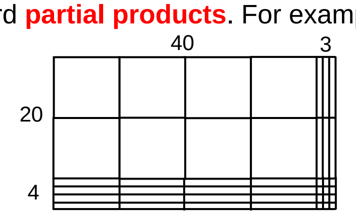
Strategies for multiplication can be more complex than those for addition and subtraction. Students need to be flexible in the way they think about the factors, and should be thinking about numbers, not just digits. Students should have many opportunities to share their ideas and practice strategies.

Model multiplying two 2-digit numbers concretely:
- Model the product as the **area** of a rectangle with the dimensions of the two numbers. This can be done using base-ten blocks and grid paper. Students should relate the model to an **algorithm**. The symbolic steps should be recorded and related to each physical manipulation.
- When the students understand the **area model**, they may choose to use a grid-paper drawing as an explanation, but it is important to record the process symbolically. A standard algorithm might be presented, but it is important that an explanation with models be provided, not just procedural rules.

The **commutative property** of multiplication means the order in which you multiply does not matter. This is sometimes helpful in rearranging the factors to make a calculation "friendlier". The **distributive property** of multiplication allows students to record **partial products**. For example:

$$43 \times 24 = (40 + 3) \times (20 + 4)$$

$$\left.\begin{array}{r} 40 \times 20 \\ 40 \times 4 \\ 3 \times 20 \\ 3 \times 4 \end{array}\right\} \text{add the products}$$



The above example closely resembles what students may already think of as **front-end multiplication**. Students should be given the opportunity to use various algorithms. If, however, students are using inefficient algorithms, they should be guided to select more appropriate ones. It is helpful for students to be exposed to various algorithms and for them to invent their own strategies. One algorithm may be more meaningful to a student than another or one algorithm may work better for a particular set of numbers. Students should be able to explain any algorithm they choose using correct mathematical language.

As for all computational questions, students should **estimate** before or after calculating. **Immediate recall of basic multiplication facts is a necessary prerequisite not only for paper-and-pencil algorithmic procedures, but also for estimation and mental computation.**

SCO: **N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Illustrate partial products in expanded notation for both factors, e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2).
- Represent both 2-digit factors in expanded notation to illustrate the distributive property, e.g., to determine the partial products of 36 × 42:
  = (30 + 6) × (40 + 2)
  = (30 × 40) + (30 × 2) + (6 × 40) + (6 × 2)
  = 1200 + 60 + 240 + 12
  = 1512
- Model the steps for multiplying 2-digit factors using an array and base ten blocks, and record the process symbolically.
- Describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model.
- Solve a given multiplication problem in context using personal strategies and record the process.

SCO: **N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Model multiplication concretely (base ten blocks, grid paper).
- Use place value language (e.g., 24 × 62 is twenty × 62 + four × 62).
- Use the language of multiplication, such as factor, product, distributive and commutative property. Effective communication of mathematical thinking should be done using words, pictures and numbers. These should be logically outlined and clearly presented in students' responses.
- Have students estimate the product first or after the calculation to judge the reasonableness of the product.
- Develop the symbolic representation from the model.
- Encourage frequent use of mental math strategies.
- Guide students to use efficient strategies to perform calculations.
- Introduce traditional algorithms only after students have a conceptual understanding of multiplication.

Suggested Activities

- Provide students with a large rectangle (e.g., 24 cm × 13 cm). Have students fill the rectangle with base ten materials to find the area. Have them write the related multiplication equation.
- Use known facts and combinations of facts that students know and apply them to more complex computations. For example, provide students with 31 × 24 and use 31 × 20, 31 × 4 and 30 × 24, 1 × 24 or other strategies to solve. Discuss which approach did they prefer and why.
- Ask students to explore the pattern in these products: 15 × 15, 25 × 25, 35 × 35, etc. Have them describe the pattern and tell how the pattern could be used to predict 85 × 85 or 135 × 135. They might then test their predictions using a calculator. Alternatively, students might explore the pattern in these products: 19 × 21, 29 × 31, 39 × 41, and use it to make a prediction for 79 × 81 and 109 × 111.
- Find the product of 25 × 25. How can the product of 25 × 25 be used to help find the products of 25 × 24, 25 × 50, and 25 × 75?
- Have students solve problems that involve 2-digit × 2-digit multiplication and are relevant to their context. For example, all 27 students in the class each brought in $18 to help pay for a field trip. How much money should the teacher have collected if everyone brought in their money? Students should be given opportunity to create and solve their own and other students' problems.
- Discuss multiplication strategies. Have students share which strategies they prefer for particular situations and why.
- Ask students to explore the following: 24 + 35 is the same as 25 + 34. Is 24 × 35 the same as 25 × 34? Have students provide an explanation.

Possible Models: base ten blocks, grid paper, calculators

SCO: **N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have the student use a model to show how to find the total money collected for photos if 43 students each bring in $23.
- Ask the students to explain why the product of two different 2-digit numbers is always greater than 100.
- Have students draw an array to show 32 × 16. Use the array to determine the product. Record the steps symbolically.
- Tell students that hardcover books were being sold at a book sale for $26. If 48 hardcover books were bought, how much money was spent?
- Ask students how far a cheetah can run in 1 minute if it runs 29 m per second. Have students explain their strategy for solving the problem.
- Prepare a series of 2-digit by 2-digit products and have students fill in missing numbers and provide justification for their choices. For example:

$$\begin{aligned} 74 \times 32 &= (70 + 4) \times (\_\_\_ + 2) \\ &= (70 \times 30) + (\_\_\_ \times 2) + (4 \times 30) + (4 \times \_\_\_) \\ &= 2100 + 140 + \_\_\_\_ + \_\_\_\_ \\ &= \_\_\_\_\_ \end{aligned}$$

- Show students the following:

$$\begin{array}{r} 41 \\ \times\ \underline{24} \\ 164 \\ \underline{82} \\ 246 \end{array}$$

  Ask students to explain the error and how to fix it.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N6: Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: using personal strategies for dividing with and without concrete materials; estimating quotients; relating division to multiplication.	**N6** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.	**N8** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Division problems can involve either sharing or finding the number of groups. Students should be given opportunities to solve both types of problems and explore their own strategies for solving them. Two possible strategies are described below. Using models help students visualize and think through the process. Students should know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.

Sharing Model with written record
"Make 100 sets of 3, using 300; 153 left. Make 50 sets of 3 using 150; 3 left…etc" The number of sets at each stage tends to be a multiple of 10 or 100 to facilitate computation.

$$\begin{array}{r|r} 151 & \\ 3\overline{)453} & \\ -300 & 100 \\ 153 & \\ -150 & 50 \\ 3 & 1 \end{array}$$

Distributive Property (partition numbers)
453 ÷ 3 =
Think 453 = 300 + 150 + 3
(300 ÷ 3) + (150 ÷ 3) + (3 ÷ 3)
100 + 50 + 1 = 151

Division should be connected to multiplication and **estimation** to check the reasonableness of the answer. Division can be modelled as "**equal sharing**" using base ten blocks. Recording the process symbolically will support student's understanding of division. The traditional long-division algorithm, whether modelled with base ten blocks or not, is best described using "sharing words" (e.g., "4 hundreds shared among 3, each gets 1 with 1 hundred left. Trade 1 hundred for 10 tens; now 15 tens to share, each gets 5 tens, etc.").

When dividing whole numbers, there are often remainders. Students must understand what these remainders mean as well as how to express them symbolically. The context of **remainders** must be discussed with students. They must understand why the number of units leftover after the sharing must be less than the **divisor**. Models help to clarify this idea. Students need many opportunities to explore the different interpretations of the remainder in problem solving situations to decide if it should be **ignored**, **rounded up**, expressed as a **fraction** or a **decimal**. A common mistake of students is to write a remainder as a decimal when the divisor is not 10 (e.g., a remainder of 7 is written as ".7"). This should be addressed through a discussion of remainders and the meaning of tenths.

Students should have many opportunities to solve and create problems which are relevant to them. These opportunities provide students with a chance to practice their computational skills and clarify their mathematical thinking.

SCO: **N6: Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model the division process as equal sharing using base ten blocks (or other model) and record it symbolically.
- Model the division process as finding how many equal groups using base ten blocks (or other model) and record it symbolically.
- Explain that the interpretation of a remainder depends on the context:
  - round up the quotient, e.g., the number of five passenger cars required to transport 13 people
  - ignore the remainder, e.g., making teams of 4 from 22 people; 2 people left over
  - express remainders as fractions, e.g., five apples shared by two people
  - express remainders as decimals, e.g., measurement and money.
- Solve a given division problem in context using personal strategies and record the process.

SCO: **N6: Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

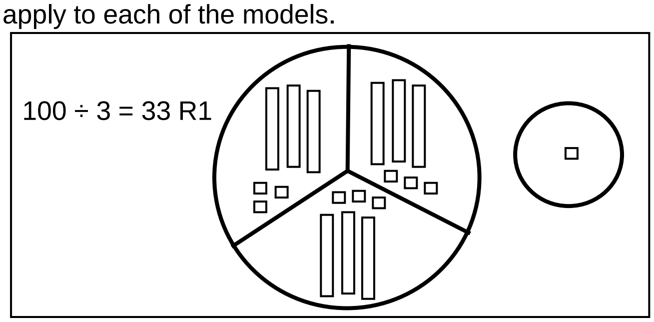
Choosing Instructional Strategies

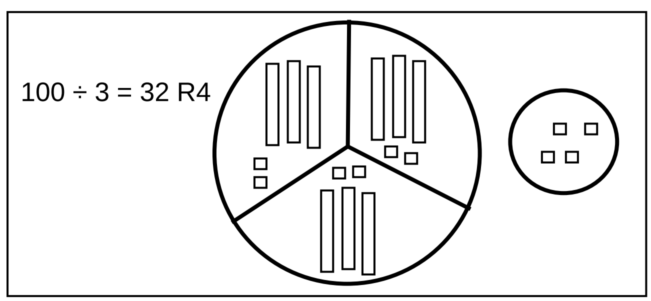
Consider the following strategies when planning lessons:

- Provide students the opportunity to solve division problems using base ten materials and other materials.
- Provide students with the opportunity to partition numbers to solve problems (e.g., for $92 \div 4$. think $92 = 80 + 12 \rightarrow 80 \div 4 = 20$ and $12 \div 4 = 3$, so the quotient is 23).
- Present division questions in a problem solving situation.
- Provide regular practice and discussion of estimation strategies to support division.
- Use children's literature, such as *One Hundred Hungry Ants* by E. Pinczes, to explore the different ways that a quantity can be put in equal groups.
- Have students create, share, and solve problems involving division.
- Use multiplication to help estimate and solve division questions. For example to solve $448 \div 7$, think how many groups of 7 would be close to 448. Sixty groups would be 420 and seventy groups would be 490 which is more than 448. The quotient must be between 60 and 70. Since 448 is 28 more than 420, it would be possible to make 4 more groups and the quotient would be 64.

Suggested Activities

- Ask students to write a word problem involving division where their interpretation of the remainder would be:
  - a situation in which the remainder would be ignored;
  - a situation in which the remainder would be rounded up; and
  - a situation in which the remainder would be part of the answer.
- Tell students that a scientist discovered a group of creatures in the Bay of Fundy. The total number of legs was 84. If each creature had the same number of legs, how many were there and how many legs were on each? Give three different possibilities and explain. Use words and pictures in your explanation.
- Ask the student to tell what division is being modelled below and to provide a word problem that would apply to each of the models.





Possible Models: base ten blocks, linking cubes, counters, money

SCO: **N6: Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask the student to use materials to model how to divide 489 by 7.
- Tell students that Simon solved the following problem: "There were 115 people going to a soccer game. Each mini bus can carry 8 people. How many mini buses are needed?"
  Simon's final answer was 15. Explain.
- Have students create and solve a problem involving division with a divisor of 6 and a dividend of 252.
- Tell students that at the "T-Shirt Shop", you can buy t-shirts in packages of 8. One package costs $130. At "Big Deals", a t-shirt costs $18. Does "Big Deals" have the better price? How do you know? Have students record and explain their process.
- Tell students that Jenna solved a problem by dividing 288 by 4. She said the answer was 72. What could the problem have been?
- Tell students that Lee is a farmer. He has 324 metres of fencing material to build a new space for his animals. He wants each side of the space to be the same length. What are three different possible spaces you would recommend that he make? How many sides would there be in each one and how much fencing would be left over?
- Ask student in which of the following situations would you
  a. ignore the remainder
  b. round up the quotient
  c. express as a fraction.
  Explain.
  i. William has 185 hockey cards that he wants to share equally among his three friends. How many cards will each person receive?
  ii. Mrs. Cormier has 9 bars of chocolate to share equally among her 4 nephews. How much chocolate will each nephew receive?
  iii. Sean can transport 3 people in his canoe. How many trips would take him to transport 35 people across a river?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to:** **• create sets of equivalent fractions** **• compare fractions with like and unlike denominators.** [C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: name and record fractions for the parts of a whole or a set; compare and order fractions; model and explain that for different wholes, two identical fractions may not represent the same quantity; provide examples of where fractions are used.	**N7** Demonstrate an understanding of fractions by using concrete and pictorial representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.	**N4** Relate improper fractions to mixed numbers.

ELABORATION

Guiding Questions:
• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Developing number sense with fractions takes time and is best supported with a conceptual approach and the use of materials. Using a variety of manipulatives helps students understand properties of fractions and focus on the relationship between the two numbers in a fraction. It is important for students to understand that fractions do not indicate anything about the size of the whole they are describing.

Students should continue to use conceptual methods to compare fractions. These methods include:

- comparing each to a **benchmark** (e.g., is $\frac{2}{5}$ more than or less than $\frac{1}{2}$);
- comparing the two **numerators** when the fractions have the same denominator;
- comparing the two **denominators** when the fractions have the same numerator. A common error made by students at this level is to think because of their experience comparing whole numbers that a larger denominator means the fraction is larger (e.g., they think $\frac{4}{7}$ is greater than $\frac{4}{6}$).

Considerable time needs to be spent on activities and discussions to develop a strong number sense of fractions. Provide students with a variety of experiences using different models (number lines, pattern blocks, counters, etc.) and different representations of the whole with the same model. Students should recognize that a fraction can **name part of a set** as well as **part of a whole** and the size of these can change. Students also need to understand that fractions can only be compared if they are parts of the same whole. Half of a cake cannot be compared to half of a brownie. When comparing one half and one quarter, the whole is the “unit” (1).

It is important that students are able to **visualize equivalent fractions** as the naming of the same **region** or **set** partitioned in different ways. Students should be given opportunities to explore and develop their own strategies for creating equivalent fractions. They should be able to explain their strategy to others. Rules for multiplying numerators and denominators to form equivalent fractions should not be provided to students to follow without a conceptual understanding of why they work.

SCO: **N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Create a set of equivalent fractions and explain why there are many equivalent fractions for any given fraction using concrete materials.
- Model and explain that equivalent fractions represent the same quantity.
- Determine if two given fractions are equivalent using concrete materials or pictorial representations.
- Formulate and verify a personal strategy for developing a set of equivalent fractions.
- Identify equivalent fractions for a given fraction.
- Compare two given fractions with unlike denominators and explain the strategy.
- Position a given set of fractions with like and unlike denominators on a number line and explain strategies used to determine the order.

SCO: **N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

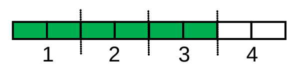
Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

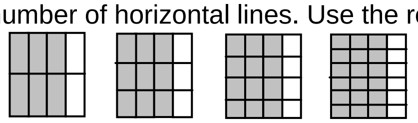
Choosing Instructional Strategies

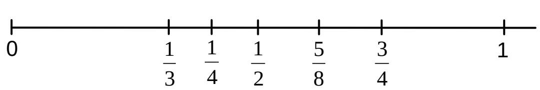
Consider the following strategies when planning lessons:

- Provide the students with a variety of activities that include the three interpretations of fractions: 1) part of a whole (e.g., part a chocolate bar); 2) part of a set (e.g., part of 30 marbles); and 3) part of a linear measurement (e.g., part of a 4 m piece of rope).
- Provide many opportunities for students to model fractions both concretely and pictorially, using a variety of models such as, pattern blocks, grid paper, fraction pieces, fraction towers, counters, Cuisenaire® rods, egg cartons, number lines, etc.
- Point out to the student that to rename $\frac{6}{8}$ as $\frac{3}{4}$, you can "clump" the 8 sections of the whole into twos. There are then four groups of 2 sections; three of the four groups are shaded. 

- Use number lines and other models to compare fractions and explore equivalencies.
- Use children's literature, such as *Fraction Action* by Loreen Leedy, to review basic fraction concepts.

Suggested Activities

- Fold a piece of paper into fourths. Colour $\frac{1}{4}$. Fold the paper again. What equivalent fraction is represented? Fold the paper again. What equivalent fraction is shown? Discuss the pattern.
- Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.
- Give students a sheet with 4 squares. Have them shade $\frac{3}{4}$ on each square vertically. Have them subdivide each square with a different number of horizontal lines. Use the resulting pictures to find possible equivalent fractions for $\frac{3}{4}$. 
- Provide students with a number line that has one of the fractions placed incorrectly. Have students identify the error and provide an explanation for where it should be correctly placed.



Possible Models: grid paper, number lines, double number lines, fraction pieces, Cuisenaire® rods, counters, egg cartons, pattern blocks, geoboards, colour tiles, fraction circles, dominoes

SCO: **N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

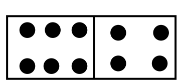
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

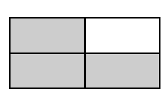
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students create a diagram or use a model to show why $\frac{4}{8} = \frac{1}{2}$ are equivalent.
- Have students explain the meaning of equivalent fractions using words, numbers and pictures.
- Provide students with a set of equivalent fractions, such as $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}, \frac{8}{24}, \frac{16}{48}$. Have students describe a pattern for the set of fractions.
- Ask the student to use his/her fingers and hands to show that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions. Alternatively, the student might be asked to choose a different model or manipulative to show this or another equivalence.
- Have students place the following fractions on a number line: $\frac{1}{2}, \frac{9}{10}, \frac{4}{5}, \frac{1}{5}$. Explain why the strategy they used to determine the location of each fraction.
- Have students make a diagram and identify the "clump size" that should be used to show that $\frac{10}{15} = \frac{2}{3}$. Ask how one might predict the "clump size" without drawing the diagram.
- Have students select a domino and write the fraction it represents. Write 2 equivalent fractions for it.

 $\Rightarrow \frac{4}{6} = \frac{2}{3} = \frac{8}{12}$

- Have students write two equivalent fractions for the following diagram. Show their work pictorially and symbolically.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

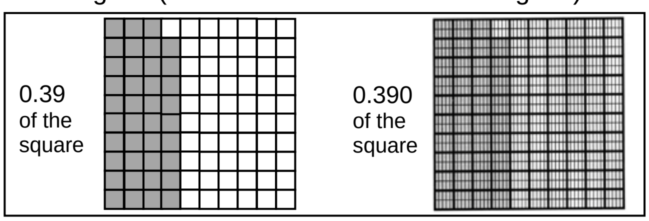
Scope and Sequence of Outcomes

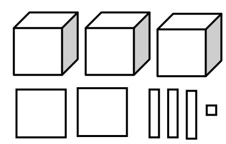
Grade Four	Grade Five	Grade Six
N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.	**N8** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.	**N1** Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students need to understand that anything to the right of the decimal represents a quantity less than one. Students should continue to use physical materials to represent or model decimals. In this way, they can better see the relationship between **hundredths** and **thousandths**. For example, students might use a thousandths grids (the same size as hundredths grids) to model decimals to the thousandths.




3.231

Alternatively, base ten blocks might be used to illustrate the relationship. Within a given context, the large block could represent 1, and then the flat would represent 0.1, the rod 0.01 and the small cube 0.001. The model for 3.231 could be modelled as shown above. It is helpful to vary which block represents one whole, so students develop flexibility in their thinking about decimal fractions. Students sometimes struggle with the concept that thousandths are smaller than tenths and hundredths based on their previous knowledge that thousands are larger than tens and hundreds. It is important for students to recognize that decimals extend the place value system to represent the parts of a whole. While money is a commonly used representation for decimals, keep in mind that it typically only represents tenths and hundredths. Students do not usually think about the cents as being part of the whole. Students can represent thousandths using **length measurements**, since 1 mm = 0.001 m. For example, 0.423 m can be represented as 423 mm, 42.3 cm (a little more than 42 cm).

Like fractions, decimals have multiple names and students must become proficient at representing, naming, and identifying equivalent decimals (e.g., 5.67 could be read as “five and sixty-seven hundredths” or “fifty-six tenths and 7 hundredths”). Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions (SCO N9). For example, 3.147 should be read as “three and one hundred forty-seven thousandths” not “three point one four seven”.

Students should recognize that thousandths can represent something quite small or something very large. For example, 0.025 m is only 2.5 cm, which is a small measurement; however, 0.025 of the population of Canada refers to 25 out of every thousand people, and 25 000 of every million people, or a very large number of people. Discussions such as these help students to develop greater number sense.

SCO: **N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
- Represent an equivalent tenth, hundredth or thousandth for a given decimal using a grid.
- Express a given tenth as an equivalent hundredth and thousandth.
- Express a given hundredth as an equivalent thousandth.
- Describe the value of each digit in a given decimal.

SCO: **N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Write decimals using place value language and expanded notation to help explain equivalence of decimals.
  0.4 = 4 tenths
  0.40 = 4 tenths + 0 hundredths
  0.400 = 4 tenths + 0 hundredths + 0 thousandths
  Since adding zeros has no effect, 0.4 must equal 0.40 and 0.400
- Use the same sized tenth, hundredths and thousandths grid squares to draw equivalent decimals.
- Help students extend the place value system to decimals by focusing on the basic pattern of ten. While building on their understanding of tenths and hundredths from Grade 4, students need to know that it takes 1000 equal parts (thousandths) to make one whole. Explore the pattern of the place value names (whole numbers and decimals).
- Vary the representation of the whole. Use a cube, flat, and rod to represent the whole in different situations. Students often have a fixed notion of what these models represent and it is important to reinforce the idea that a decimal relates a part to a whole the same way that fractions do.

Suggested Activities

- Present a riddle to the class such as, “I have 25 hundredths and 4 tenths. What am I?” Have students use a model of their choice to represent the solution to the riddle.
- Make sets of cards showing decimals in different forms including expanded form, pictorial representations and equivalent decimals. Students can play matching games or decimal snap.
- Provide opportunities for students to find and share how large numbers are represented in newspapers and magazines. For example, an executive’s salary may be written as 4.5 million dollars.
- Place five different displays of combinations of base ten blocks. Ask the students to visit the centre and record the five decimals displayed.
- Provide students with two hundredths circles, each of a different color. Cut each disk along one radius so they can be fit together. Students can use these to model given decimals, or to write decimals from a given model.
- Use the calculator to “count”. Enter 0.1 + 0.1 =, + 0.1 =, =, =… when the display shows 0.9 have students predict what number will be next. Extend this to use 0.01 and 0.001 to demonstrate the relative magnitude of hundredths and thousandths.
- Ask the student to identify a situation in which 0.750 represents a large amount and one in which it represents a small amount (e.g., 0.750 of a million dollars; 0.750 of a dollar).

Possible Models: base ten blocks, number lines, hundredths circle, hundredth and thousandth grids

SCO: **N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

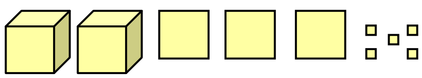
Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask the student to express 0.135 at least three different ways (e.g., one tenth, three hundredths, five thousandths; thirteen hundredths, five thousandths; one hundred thirty-five thousandths).
- Tell the student that gasoline is priced at 83.9¢ per litre. Ask: What part of a dollar is this?
- Ask the students to write 10 different decimal numbers that have tenths, hundredths and/or thousandths. Have them draw pictures of base ten blocks that would represent their numbers.
- Present student with a base ten model of decimal numbers and ask the student to represent the model with a decimal number.



- Ask students use hundredths and thousandths grids or base ten blocks to model equivalent decimals.
- Show the student cards on which decimals have been written (e.g., 0.4 m, 0.75 m and 0.265 m). Ask the student to place the cards on a metre stick at the correct location.
- Give students a drawing of an irregular shape and have them shade in approximately 0.247 of it.
- Ask the student to write the numerals for "two hundred fifty-six thousandths" and "two hundred and fifty-six thousandths". Ask the student to explain why watching and listening for "and" is important when interpreting numbers.
- Give students three number cubes. Have them make the greatest and least possible decimals using the numbers rolled as the digits. Have students read the decimal numbers aloud.
- Have students describe the meaning of each digit in a given decimal (e.g., 6.083).

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N9: Relate decimals to fractions (to thousandths).**
[CN, R, V]
N10: Compare and order decimals (to thousandths) by using:
- **benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
N10 Relate decimals to fractions (to hundredths).	**N9** Relate decimals to fractions and fractions to decimals (to thousandths). **N10** Compare and order decimals (to thousandths), by using: • benchmarks • place value • equivalent decimals.	**N1** Demonstrate an understanding of place value for numbers: greater than one million; less than one thousandth.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Decimals are another way of writing fractions. Students should continue to build their conceptual understanding of the relationship of decimals to fractions as they explore numbers to the **thousandths**. One thousandth (0.001) can be written as $\frac{1}{1000}$. Students should be encouraged to read decimals as fractions (e.g., 0.246 is read as 246 thousandths and can be written as $\frac{246}{1000}$). Measurement contexts provide valuable learning experiences for decimal numbers because any measurement can be written in an equivalent unit that requires decimals (e.g., one metre is $\frac{1}{1000}$ of a kilometre, 1 m = 0.001 km).
To develop decimal and fractional number sense, it is essential to discuss the **magnitude** of the number, such as 493 thousandths is about one half, and 1.761 is about $1\frac{3}{4}$. Using number lines with benchmarks such as $\frac{1}{4}$ (0.25), $\frac{1}{2}$ (0.5), $\frac{3}{4}$ and (0.75) is helpful to create a visual reference for students.
Students should be able to determine which of two decimal numbers is greater by comparing the whole number parts first and then the amounts to the right of the decimals. It is important that students understand that decimal numbers do not need the same number of digits after the decimal to be compared. For example, it can quickly concluded that 0.8 > 0.423, without converting 0.8 to 0.800, because the former is much more than half (a benchmark) and the latter is less than half. A common misconception is students may think that 0.101 is greater than 0.11 because 101 is larger than 11. Others may think 0.101 is less because it has a digit in the thousandths place, while the other number has only hundredths. These same students may say 0.101 is less than 0.1 because it has thousandths while 0.1 has only tenths. Such misconceptions can be dealt with by having students create representations of the numbers that are being compared using models. It is helpful to use place value or equivalent decimals to compare and order these numbers. Students should be given opportunity to explore connections between models and oral and written forms. It is also beneficial to examine the connection is made between decimals and base ten fractions to understand **decimal equivalence** (e.g., 0.3 = $\frac{3}{10}$ or $\frac{30}{100}$ or $\frac{300}{1000}$).

SCO: **N9: Relate decimals to fractions (to thousandths).**
[CN, R, V]
N10: Compare and order decimals (to thousandths) by using:
- **benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

N9
- Write a given decimal in fractional form.
- Write a given fraction with a denominator of 10, 100 or 1000 as a decimal.
- Express a given pictorial or concrete representation as a fraction or decimal, e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$.

N10
- Order a given set of decimals by placing them on a number line that contains benchmarks, 0.0, 0.5, and 1.0.
- Order a given set of decimals including only tenths using place value.
- Order a given set of decimals including only hundredths using place value.
- Order a given set of decimals including only thousandths using place value.
- Explain what is the same and what is different about 0.2, 0.20 and 0.200.
- Order a given set of decimals including tenths, hundredths and thousandths.

SCO: **N9: Relate decimals to fractions (to thousandths).**
[CN, R, V]
N10: Compare and order decimals (to thousandths) by using:
- **benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

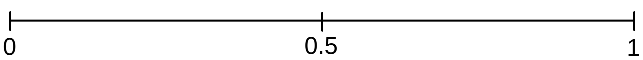
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students place only decimal tenths on a number line, then repeat for decimal hundredths and thousandths.



- Have students use thousandth grids to model the equivalency of tenths, hundredths and thousandths (e.g., 0.3, 0.30, 0.300) and explain what is the same and what is different.
- Have students order a given set of decimals including tenths, hundredths and thousandths using equivalent decimals. For example, to order 0.402, 0.39 and 0.7, students could think of them as thousandths (e.g., 0.402, 0.390, 0.700).
- Have students begin to explore the relationship between fraction and decimal benchmarks. For example, 0.5 is another name for $\frac{1}{2}$; 0.25 is another name for $\frac{1}{4}$; 0.75 is another name for $\frac{3}{4}$.
- Represent decimals in a variety of ways. For example: 0.452 is $\frac{452}{1000}$ and can be expressed as 0.4 + 0.05 + 0.002 or $\frac{4}{10} + \frac{5}{100} + \frac{2}{1000}$.
- Provide a variety of models, stressing the magnitude of the number. For example, 0.452 could be modelled using a number line (about one half), base ten blocks, thousandths grids, place value chart.

Suggested Activities

- Have students express given numbers as fractions and decimals (e.g., sixty-four hundredths, $\frac{64}{100}$, 0.64).
- Ask students to investigate where in the media fractions and decimals are used, and to write a report on their findings.
- Give students a "number of the day" and have them express this number in as many ways as they can. For example: 0.752 could be shown as: $\frac{752}{1000}$ or $\frac{7}{10} + \frac{5}{100} + \frac{2}{1000}$ or about $\frac{3}{4}$; plotted on a number line; modelled with base ten materials on a place value chart; shown on a thousandths grid; or described in a variety of ways ("It's 0.248 less than one whole", etc.).
- Give each student a different irregular shape and ask him/her to tear off about 0.256 of that shape. Have students explain how they estimated 0.256, and why pieces may not be the same size or shape.

Possible Models: base ten blocks, thousandth grids, number lines, place value charts

SCO: **N9: Relate decimals to fractions (to thousandths).**
[CN, R, V]
N10: Compare and order decimals (to thousandths) by using:
- **benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

ASSESSMENT STRATEGIES

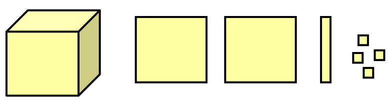
Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students compare and order decimals tenths, hundredths and thousandths and express them as fractions.
- Have students model decimal thousandths using base ten blocks.
  For example: 1.214



- Have students place decimals and fractions on a number line, such as: $\frac{3}{4}$, 0.31, $\frac{6}{10}$, $\frac{102}{1000}$.
- Tell students that they have properly placed 796 pieces of the 1000-piece jigsaw puzzle. Ask what part (fractional and decimal) of the puzzle has been completed? What part of the puzzle has yet to be finished? ($\frac{204}{1000}$, 0.204)
- Have students continue the following series, counting by tenths.
  0.5, 0.6, 0.7, ____ , ____ , ____ , ____

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V, ME]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

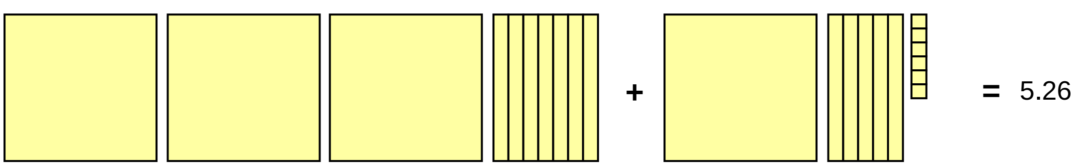
Grade Four	Grade Five	Grade Six
N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: using compatible numbers; estimating sums and differences; using mental math strategies to solve problems.	**N11** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).	**N8** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

It is essential that students recognize that all of the properties and developed strategies for the addition and subtraction of whole numbers also apply to decimals. For example, adding or subtracting **tenths** (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). This could be extended to addition with tenths that total more than one whole (e.g., 7 tenths and 4 tenths are 11 tenths or 1 and 1 tenth). The same is true with **hundredths** and **thousandths**. Rather than simply telling students to line up decimals vertically, or suggesting that they "add zeroes," they should be directed to think about what each **digit** represents and what parts go together. For example, to add 1.625 and 0.34, a student might think using front end addition, 1 whole, 9 (6 + 3) tenths and 6 (2 + 4) hundredths, and 5 thousandths or 1.965.

Base ten blocks and hundred grids are useful models to represent addition with decimals up to hundredths. If a flat represents one whole unit, then 3.7 + 1.56 would be modeled as:



Students need to recognize that **estimation** is a useful skill when doing computations of whole and decimal numbers. Estimation can be used to determine if the sum or difference is reasonable and with decimal placement. To be efficient when estimating **sums** and **differences** mentally, students need a variety of strategies from which to choose so they can select one that is efficient for the numbers involved. For example, a student may use front-end estimation to add 9.35 + 8.106. They would estimate each decimal to the nearest whole number (9 + 8) and know that the sum is greater than 17. Ensure that students have sufficient practice with a variety of mental math strategies so that these acquired skills can be readily applied to solve various problems. When a problem requires an exact answer, students should first look at the numbers to determine if they are able to calculate the answer mentally. If no mental math strategy is efficient for the numbers involved, then the student can explore which other strategy is best to use.

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V, ME]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Place the decimal point in a sum or difference using front-end estimation, e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312.
- Correct errors of decimal point placements in sums and differences without using paper and pencil.
- Explain why keeping track of place value positions is important when adding and subtracting decimals.
- Predict sums and differences of decimals using estimation strategies.
- Solve a given problem that involves addition and subtraction of decimals, limited to thousandths.

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V, ME]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide opportunity for students to model and solve addition and subtraction questions involving tenths, hundredths and thousandths concretely, pictorially, and symbolically (e.g., thousandths and hundredths grids, base ten blocks, and number lines).
- Present addition and subtraction questions both horizontally and vertically to encourage alternative computational strategies. For example, for 1.234 + 1.990, students might calculate:
  1.234 + 2 = 3.234 followed by 3.234 - 0.01 = 3.224.
- Have students investigate the relationship between adding decimals numbers and whole numbers. For example, 356 + 232 = 588; this looks similar to 0.356 + 0.232 = 0.588.
- Provide problem solving situations that require students to add or subtract decimals using a variety of strategies.
- Have students determine an estimate when solving problems involving the addition and/or subtraction of decimals.

Suggested Activities

- Provide base ten blocks or thousandths grids. Give the student addition or subtraction questions (decimal numbers) to represent using models. Be sure to include questions that require regrouping.
- Model 4.23 and 1.359 with base ten blocks or thousandth grids. Ask students to use the materials to explain how to find the difference between the two numbers.
- Provide students with the batting averages of some baseball players. Have them calculate the spread between the player with the highest average and the one with the lowest. Have students create problems using the averages on the list.
- Request that the students provide examples of questions in which two decimal numbers are added and the answers are whole numbers.
- Have students create and solve their own and each other's word problems in a context that is relevant to them.
- Tell students that you have added three numbers, each less than 1, and the result is 2.4. Ask if all the decimal numbers could be less than one half and to explain why or why not. Once students realize the numbers cannot all be less than one half, ask them how many could be less.
- Have students find situations in which decimals are added and subtracted beyond their classroom experiences and present their findings to the class.

Possible Models: base ten blocks, hundredth and thousandth grids, number lines

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V, ME]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask the students to fill in the boxes so that the answer for each question is 0.4. The only restriction is that the digit 0 cannot be used to the right of the decimal points.

$$\begin{array}{r} \square.\square\square \\ +\ \square.\square\square \\ \hline \end{array} \qquad \begin{array}{r} \square.\square\square \\ -\ \square.\square\square \\ \hline \end{array} \qquad \square.\square\square + \square.\square\square =$$

- Present the following situation in which Juli made an error when she subtracted. Ask the students what could be said to Juli to help her understand why the answer is incorrect: 5.23 - 1.453 = 3.783.
- Ask students to use a model to explain how to find the sum and difference of two decimal numbers.
- Provide students with addition and subtraction questions in which the decimal is missing from the sum or difference. Have students place the decimal in the correct position in each answer.
- Tell students that Tim added 2.542 + 13.6 and said that the sum was 16.142. Jake added the same numbers and said the answer was 2.678. Explain why the answers are different. Who is right? How do you know?
- Use an example to explain why it is important to keep track of place value positions when adding and subtracting decimal numbers. (This can be written in a journal.)
- Have students solve problems such as:
  - John needs 2 kg of hamburger for a recipe. He has a 0.750 kg package. How much more does he need to buy?
  - Sasha bought two books at the book fair. One was $6.95 and the other was $7.38. How much change will she get from a $20 bill?

  Ensure that students estimate as part of the problem solving process.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR1: Determine the pattern rule to make predictions about subsequent terms (elements).** [C, CN, PS, R, V]
[C] Communication **[T]** Technology · **[PS]** Problem Solving **[V]** Visualization · **[CN]** Connections **[R]** Reasoning · **[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
PR1 Identify and describe patterns found in tables and charts, including a multiplication chart.	**PR1** Determine the pattern rule to make predictions about subsequent terms (elements).	**PR1** Demonstrate an understanding of the relationship within tables of values to solve problems. **PR2** Represent and describe patterns and relationships using graphs and tables.

ELABORATION

Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Patterns are foundational to understanding many mathematical concepts. The ability to create, recognize and extend patterns is essential for making generalizations, seeing relationships, and understanding the order and logic of mathematics (Burns, 2007; p.144). These skills provide the groundwork for **algebraic reasoning** and inquiry.

Patterns represent identified regularities based on rules describing the patterns' **terms (elements)**. Unless a pattern rule is provided there is no single way to extend a pattern (e.g., 1, 3, 5, 7 might be an odd number sequence, or a repeating sequence 1, 3, 5, 7, 1, 3, 5, 7…). In Grade 5, students build on previous knowledge of growing and decreasing patterns to focus on describing these patterns and their relationships mathematically. They will use this knowledge to make and verify predictions of missing elements in various patterns.

Patterns can be used to represent a situation and to solve problems. They can be **extended** with and without concrete materials and can be described using mathematical language. When discussing a pattern, students should be encouraged to determine how each step in the pattern is different from the preceding step.

				xxx					
			xxx	xxx					
		xxx	xxx	xxx					
	xxx	xxx	xxx	xxx					
Step	1	2	3	4	5	6	?	…	20
Number of X's	3	6	9	12	?	?	?	…	?

Tables and charts provide an opportunity to display patterns and see relationships. For most students, these tables and charts make it easier to see the patterns from one step to the next. When a chart has been constructed, the differences from one step to the next can be written by it. Students will probably first observe the pattern from one step to the next; however, using the chart to find the twentieth or hundredth step is not reasonable. If a rule or relationship can be discovered, any table entry can be determined without building or calculating all of the intermediate entries. Students will learn that the rule can be described as a **mathematical expression**. For example, in the above pattern, the rule could be described as $3 \times n$ or $3n$. Therefore the 20^{th} step would be 3×20 (60). Students should be given frequent opportunity to use materials to represent patterns and explain orally and in writing how elements in various patterns change as the patterns are extended.

SCO: **PR1: Determine the pattern rule to make predictions about subsequent terms (elements).**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Extend a given pattern with and without concrete materials, and explain how each term (element) differs from the preceding one.
- Describe, orally or in writing, a given pattern using mathematical language, such as one more, one less, five more.
- Write a mathematical expression to represent a given pattern, such as $r + 1$, $r – 1$, $r + 5$.
- Describe the relationship in a given table or chart using a mathematical expression.
- Determine and explain why a given number is or is not the next term (element) in a pattern.
- Predict subsequent terms (elements) in a given pattern.
- Solve a given problem by using a pattern rule to determine subsequent terms (elements).
- Represent a given pattern visually to verify predictions.

SCO: **PR1: Determine the pattern rule to make predictions about subsequent terms (elements).** [C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

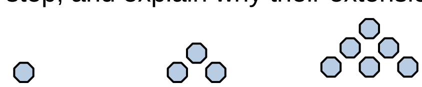
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

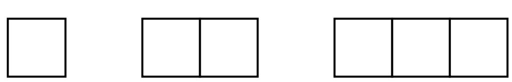
- Have students practice extending patterns with materials and drawings and then translate the pattern elements to a table or T-chart. Ask them to describe what is happening as the pattern grows or decreases and how the new step is related to the previous one.
- Have students describe, using mathematical language (e.g., one more, seven less) and symbolically (e.g., $r + 1$, $p - 7$), a pattern represented concretely, pictorially, or from a chart.
- Have students verify whether or not a particular number belongs to a given pattern.
- Provide students with opportunities to predict terms in a pattern. They should be expected to explain and verify their predictions. Visual representations, such as models or drawings are helpful.
- Have students solve problems and make decisions based upon the analysis of a pattern.

Suggested Activities

- Show students the first three or four steps of a pattern. Provide them with appropriate models and grid paper and have them extend the patterns recording each step, and explain why their extension follows the pattern. Have them determine the pattern rule.



- Have students examine number sequences to determine subsequent terms (elements) and explain their extensions. Ask students to determine the pattern rule.
  1, 4, 7, 10, 13, …   42, 36, 30, 24, 18 …   0, 2, 6, 14, 30, …
  Extension: Have students give two numbers that cannot come next and explain why.
- Ask students to work in pairs to explore the many patterns on a multiplication chart (e.g., square numbers on the diagonal, sums of rows and columns, adjacent square patterns, doubling between columns, such as the 2's, 4's, and 8's).
- Provide students with a growing pattern and have them extend it. They should make a table showing how many items are needed to make each step of the pattern. Have them predict the number of items in the tenth or twentieth step of the pattern. For example, four people can sit at one table, six people can sit at two tables pushed together, eight people can sit at three tables. How many can sit at ten tables? Twenty? How many tables are needed for 24 people?



Number of tables	1	2	3	4	…
Number of seats	4	6	8	?	…

This pattern could be displayed on a T-chart

Tables	Seats

Possible Models: counters, linking cubes, grid paper, dot paper, pattern blocks, colour tiles, multiplication chart, calculators

SCO: **PR1: Determine the pattern rule to make predictions about subsequent terms (elements).** [C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

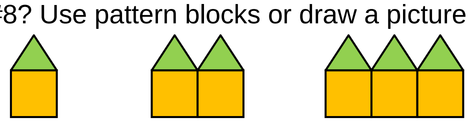
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students fill in missing terms (elements) from number sequences and identify the pattern rules.
  a. 1, 4, _____, 16, ___, 36 ($n \times n$)
  b. 18, 16, 14, _____, _____
  c. 2.4, 2.7, _____ , _____ , 3.6
- Give the students a chart showing the input and the output and ask them to provide the possible rule.

Input	Output
2	9
3	10
4	11
5	12

- Show students the picture below. The first "house" has two shapes. The second "house" has 4 shapes and the third "house" has 6 shapes. Have students predict the number of shapes in "house" #4 and "house" #8? Use pattern blocks or draw a picture of each of the eight houses to verify your answer.



- Ask students to explain whether 84 would be included in each of the following patterns:
  a. 1, 3, 5, 7……
  b. 4, 8, 12, 16 ……
  c. 200, 192, 184, 176 ……
- Have students solve real-world problems that require identifying a pattern rule to determine subsequent terms (elements). For example, to bake cookies for a school bake sale, the quantities of ingredients in the recipe must be determined for multiple batches of cookies. If 2 cups of sugar and 3 cups flour is needed for one batch, how much is needed for 4 batches? 7 batches?
- Show students a table that shows the relationship between the number of students going to a movie and the total cost of the tickets. Have students describe the relationship between the students and the cost using a mathematical expression. Use the pattern to determine the number of students at the movie if the tickets cost $98.

Students	1	2	3	4	?
Cost of tickets	$7	$14	$21	$28	$98

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

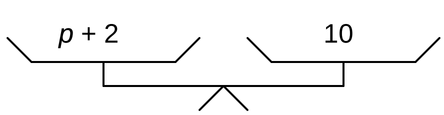
Grade Four	Grade Five	Grade Six
PR5 Express a given problem as an equation in which a symbol is used to represent an unknown number. **PR6** Solve one-step equations involving a symbol to represent an unknown number.	**PR2** Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**PR3** Represent generalizations arising from number relationships using equations with letter variables. **PR4** Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.

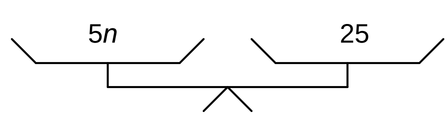
ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Exploring patterns leads to algebraic thinking. Algebra is a system that allows us to represent and explain mathematical relationships. Students are thinking algebraically when they solve open number sentences like 5 + ☐ = 13, first using boxes or open frames, then using letters, 5 + *n* = 13. Students usually progress from the use of open frame to letters. When letters are used in mathematics, they are called **variables**. It is useful for students to think of variables as numbers that can be operated on and manipulated like other numbers. An **equation** is a mathematical sentence with an equal sign. Students have been exploring the concept of equality since Grade 2. It is important for students to recognize that the **equal sign** indicates that both sides of the equation are balanced and does not simply mean "the answer is".

In order to solve an equation, we need to find the value of the variable to make the equation true. Using the balance concept on a regular basis will help students develop a visual image for solving equations.





An **expression** does not include an equal sign and is used most frequently to describe a pattern rule. A **coefficient** in elementary algebra is the numerical part of an expression usually written before the literal part (letter). In the expression, 3*b*, the 3 is the coefficient. It is the **constant** part of the **term**. A term is part of an algebraic equation or expression that may be a number, variable, or a product of both.

$\boxed{k + 6}$ "*k*" is a term and "6" is a term $\boxed{35 = 7y}$ "7" is the coefficient in the term 7*y*

SCO: **PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain the purpose of the letter variable in a given addition, subtraction, multiplication or division equation with one unknown; e.g., $36 \div n = 6$.
- Express a given pictorial or concrete representation of an equation in symbolic form.
- Express a given problem as an equation where the unknown is represented by a letter variable.
- Create a problem for a given equation with one unknown.
- Solve a given single-variable equation with the unknown in any of the terms; e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$.
- Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially or symbolically.

SCO: **PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

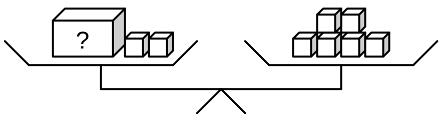
Consider the following strategies when planning lessons:

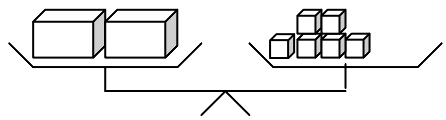
- Build on the students' knowledge from previous grades to write addition, subtraction, multiplication and division equations. Connect the concrete (use models such as counters and balance scales) with pictorial and symbolic representations consistently as the students develop and demonstrate an understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a letter to represent the unknown number.
- Have the students create problems for a variety of number sentences using the four operations.
- Explain that if the same variable, or unknown, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown; e.g., for $n + n = 20$ can be written as $2n = 20$.
- Have students complete tables such as the one below.

n	$3n$
3	
8	
	30
12	

Suggested Activities

- Have students play “Solve for my variable”.
  I subtract 6 from *n* and have 13 left. What is *n*?
  Four more than *p* is 37. What is *p*?
  Possible extension: i) Two more than 3*w* is 23. What is *w*?
  ii) One less than 4*k* is 27. What is *k*?
- Provide simple story problems and ask students to write equations. Include stories for all four operations. For example:
  - I had birthday money and spent \$6.25. I now have \$8.75 (n - \$6.25 = \$8.75 or \$6.25 + \$8.75 = n).
  - There are 3 full boxes of pencils. There are 36 pencils in all ($3a = 36$).
- Provide one-step single-variable equations and have students create story problems.
- Have students write equations for balances such as the ones below, using letters for the variables.





Possible Models: linking cubes, balance scales, counters

SCO: **PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

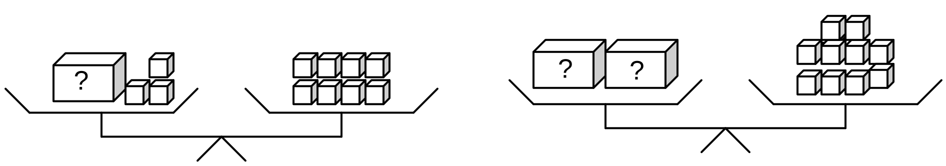
Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students draw a diagram and solve single-variable, one-step equations such as:
  $18 + n = 31$ $9 = 43 - p$ $8k = 56$ $m \div 6 = 7$

  Write word problems that could be represented by each of the above equations.
- Tell students that Nick was given the following problem to solve.
  "There were some students on the bus and 12 got off. Now there are 14 left on the bus. Now many students were originally on the bus?"
  To solve the problem, Nick wrote this equation: $b - 12 = 14$. Why did he use a letter in the equation?
- Ask students to write an equation for the following problem: There are now 15 apples in a basket. There were 24 at the start. Some have been eaten. How many apples have been eaten?
  Write an equation to represent the problem. Then solve your problem. Write another possible equation that could be written for the same problem? Explain.
- Tell students that Marci said the *w* in the following equation equals 12. Is Marci correct? Why or why not?
  $$16 = w - 4$$
- Have students write equations to describe the balance representations, such as the following:



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
SS3 Demonstrate an understanding of area of regular and irregular 2-D shapes by: recognizing that area is measured in square units**;** selecting and justifying referents for the units cm^2 or m^2**;** estimating area by using referents for cm^2 or m^2**;** determining and recording area constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.	**SS1** Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and draw conclusions.	**SS3** Develop and apply a formula for determining the: perimeter of polygons; area of rectangles; volume of right rectangular prisms.

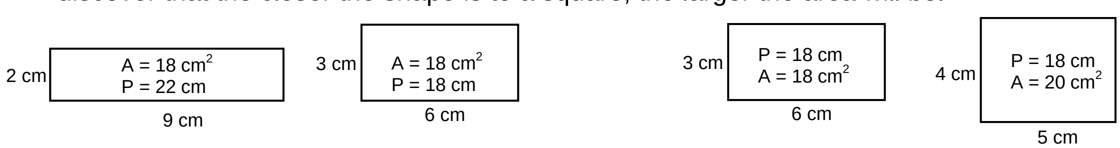
ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students in Grade 5 often do not make the distinction between area and perimeter and may calculate the area instead of perimeter or vice versa. **Area** is the measure of the space inside a region or how much it takes to cover a region. **Perimeter** is the distance around a region. It is important that students have many opportunities to construct rectangles of different areas and perimeters concretely and pictorially.

Area and perimeter involve measuring length. Rules or formulas may be invented by students as they do the activities, but formal instruction in these will take place in Grade 6. When the students are able to measure efficiently and effectively using standard units, their learning experiences can be directed to situations that encourage them to construct measurement formulas. When determining the area of a rectangle, students may realize as they count squares that it would be quicker to find the number of squares in one row and multiply this by the number of rows. When finding perimeters of rectangles, students may discover more efficient methods instead of adding all four sides to find the answer (e.g., add the length and width and double).

It is important that students learn about area and perimeter together. Through explorations, students will:
- discover that it is possible for a rectangle of a certain area to have different perimeters
- discover that is possible for rectangles with the same perimeter to have different areas.
- discover that the closer the shape is to a square, the larger the area will be.



The concepts of perimeter and area should be presented in a real-world problem solving context.

SCO: **SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Construct or draw two or more rectangles for a given perimeter in a problem-solving context.
- Construct or draw two or more rectangles for a given area in a problem-solving context.
- Illustrate that for any given perimeter, the square or shape closest to a square will result in the greatest area.
- Illustrate that for any given perimeter, the rectangle with the smallest possible width will result in the least area.
- Provide a real-life context for when it is important to consider the relationship between area and perimeter.

SCO: **SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students use geoboards to construct rectangles with specified perimeters and discuss the areas.
- Assign specific areas (e.g., 12 square units) and have students use colour tiles to create various rectangles and find the possible perimeters.
- Ask the students to use dot paper to compare the areas of rectangles with the following dimensions: 2 cm × 3 cm, 4 cm × 3 cm, 6 cm × 3 cm. Ask what they observe and have them give another set of dimensions that follows the same pattern and draw conclusions.
- Provide students with a variety of real-world contexts in which to explore the relationships between area and perimeter (e.g., flooring, fencing, playing fields, gardens, zoo enclosures, tennis courts, wallpaper, bowling alleys, etc.).

Suggested Activities

- Ask the student to explain why the perimeter of rectangles with whole number side lengths is always even. Have them use words, drawings, and/or numbers in their explanation.
- Have students relate perimeters to areas. For example, give pairs of students 24 colour tiles and ask them to find different rectangles, each with the area of 24 square units, but with different perimeters. Ask them to find a way to keep track of their rectangles and perimeters. What rectangle has the largest perimeter? The smallest? Have students draw conclusions.
- Ask the students to draw three different rectangles with the same perimeter.
- Construct rectangles on grid paper with a given perimeter, and then compare the side lengths and the areas. Have students discuss their findings and conclusions with regard to side length and area.
- Provide a 2 × 3 rectangle. Have students predict what would happen to the area and perimeter if the side lengths were doubled; or halved? Have students check their predictions and draw conclusions based on their investigation.
- Solve problems such as:
  - The cast of the latest hit movie is coming to your town. Design a “red carpet” with an area of 50 m^2 that will provide the maximum amount of room for photographers and fans to stand around its perimeter.

Possible Models: geoboards, colour tiles, dot paper, grid paper, linking cubes

SCO: **SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students compare and contrast a given pair of rectangles with the same perimeter.
  - How is a rectangle with dimensions of 3 cm by 4 cm different from a rectangle with dimensions of 2 cm by 5 cm? How are they similar?
- Ask students to choose the dimensions of the rectangle with the largest area and the smallest area from a set of rectangles with the same perimeter.
  - The following rectangles all have a perimeter of 18 cm: (1 cm by 8 cm), (2 cm by 7 cm), (3 cm by 6 cm), and (4 cm by 5 cm). Which rectangle has the largest area, smallest area?
- Have students construct (concretely or pictorially) and record the dimensions of two or more rectangles with a specified perimeter. Have students select and justify dimensions that would be most appropriate in a particular situation.
  - A rectangle is to have a perimeter of 18 units, what are the dimensions of the possible rectangles? Which rectangle would be most appropriate if the rectangle is to be the base of a shoe box or a dog pen?
- Have students construct (concretely or pictorially) and record the dimensions of as many rectangles as possible with a specified area and select, with justification, the rectangle that would be most appropriate in a particular situation.
  - A rectangle is to have an area of 24 units^2, what are the dimensions of the possible rectangles? Which rectangle would be most appropriate if the rectangle is to fence off the largest garden possible? The smallest garden possible?
- Ask students to identify situations relevant to self, family, or community where the solution to problems would require the consideration of both area and perimeter, and solve the problems.
  - The dog trainer has 22 metres of fencing to build a rectangular dog pen. Which dimensions would provide the largest play area for the dogs?
- Have students create at least two different rectangles on a geoboard with a perimeter of 12. Ask how they decided on the dimensions for the rectangles. Do all of the rectangles have the same area? Explain.
- Provide students with grid paper and have them draw at least two different rectangles with an area of 24 square units. Do all of the rectangles have the same perimeter? Explain.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS2: Demonstrate an understanding of measuring length (mm and km) by:** • **selecting and justifying referents for the units mm and km.** • **modelling and describing the relationship between mm and cm units, and between mm and m units.** • **modelling and describing the relationship between m and km units.** [C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
SS3 Demonstrate an understanding of area of regular and irregular 2-D shapes by: recognizing that area is measured in square units**;** selecting and justifying referents for the units cm^2 or m^2**;** estimating area by using referents for cm^2 or m^2**;** determining and recording area; constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.	**SS2** Demonstrate an understanding of measuring length (mm and km) by: • selecting and justifying referents for the unit mm and km • modelling and describing the relationship between mm and cm units, and between mm and m units. • modelling and describing the relationship between m and km units.	

ELABORATION

Guiding Questions:

• What do I want my students to learn?
• What do I want my students to understand and be able to do?

Measurement is fundamentally about making comparisons. At this point in their learning, students are able to compare two objects directly by accurately using standard units of length such as **centimetres** and **metres.** In Grade 5, students will extend this knowledge to include **millimetres** and **kilometres**.

Students are expected to have a **personal referent** for one millimetre and one kilometre and be able to explain their choice. They should continue to use their referents for one centimetre and one metre developed in Grade 3. Examples of referents include: one millimetre is about the thickness of a dime, one centimetre is about the width of your baby finger, one metre is about the height of the doorknob, and one kilometre is about the distance from the school to a local landmark.

In Grade 3, students have already explored the relationship between centimetres and metres. Students need to learn how to choose the appropriate unit or combination of units for the task at hand. This choice depends on the magnitude of the length to be measured and the level of precision required by the task (Small, 2008; p. 379). For example, millimetres can be used to measure small objects or to measure larger objects with more precision. Students should recognize that 1 kilometre is 1000 metres, 1 metre is 100 centimetres and 1000 millimetres, and 1 centimetre is 10 millimetres. Flexibility with using the different measurements is in the developmental stage and needs to be supported with a variety of materials and many experiences. Students need to be able to rename measurements and change from smaller units to larger units and vice versa, but also able to identify which unit is the most appropriate. For example, a pencil that is 11 cm long could also be described as 110 mm or 0.11 m.

It is important that students are encouraged to estimate measurements before actually verifying them using a measurement tool. Using rulers, metre sticks, Cuisenaire® rods and base ten blocks will provide students with benchmarks when estimating lengths.

SCO: **SS2: Demonstrate an understanding of measuring length (mm and km) by:**
- **selecting and justifying referents for the units mm and km.**
- **modelling and describing the relationship between mm and cm units, and between mm and m units.**
- **modelling and describing the relationship between m and km units.**

[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Provide a referent for one millimetre and explain the choice.
- Provide a referent for one centimetre and explain the choice.
- Provide a referent for one metre and explain the choice.
- Provide a referent for one kilometre and explain the choice.
- Show that 10 millimetres is equivalent to 1 centimetre using concrete materials, e.g., ruler.
- Show that 1000 millimetres is equivalent to 1 metre using concrete materials, e.g., metre stick.
- Provide examples of when millimetres are used as the unit of measure.
- Provide examples of when kilometres are used as the unit of measure.
- Know that 1000 metres is equal to 1 kilometre.

SCO: **SS2: Demonstrate an understanding of measuring length (mm and km) by:**
- **selecting and justifying referents for the units mm and km.**
- **modelling and describing the relationship between mm and cm units, and between mm and m units.**
- **modelling and describing the relationship between m and km units.**

[C, CN, ME, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies
Consider the following strategies when planning lessons:
- Have students choose and use referents for 1 mm, 1 cm, 1 m to determine approximate linear measurements in situations relevant to self, family, or community and explain the choice.
- Help the students develop mental images of various measurement standards. To provide estimation practice, involve students in activities such as, "Show me (with hands or arms): 75 centimetres; 20 millimetres; 0.5 metres".
- Have students use the relationships between standard metric units to rename measurements when comparing them.
- Encourage students to think of their ruler, as well as a metre stick or base ten blocks, when estimating length. Most rulers are 30 cm (or 300 mm) long and serve as good benchmarks. For example, 62 cm can be thought of as the length of about 2 rulers.
- Find a personal referent for kilometre (from the school to the post office, for example) by going on a kilometre walk. Explore how long it takes to walk this distance. As well, it is important for students to have a sense of a longer distance, such as 100 km from their home town to a nearby city.
- Use maps with scales to investigate longer distances (e.g., have students investigate distances between towns/cities in New Brunswick). Have students plan an imaginary trip to another Canadian city and determine the distance of a return trip.

Suggested Activities
- Ask the students to show, with fingers or arms, the following lengths: 550 mm, 60 cm, 0.25 m. Have them describe the length using another unit of measure.
- Ask the student to rewrite 2.3 m using other metric units.
- Ask: If you change metres to centimetres, will the numerical value become greater or less? Why?
- Have students measure objects that do not measure exact centimetres, thus stressing the importance of millimetres when striving for precision of measurement.
- Share a short paragraph describing the measurements of a variety of classroom items. Ask the students to insert the appropriate unit for each. For example: the table was 1524 ___ long. On it was a pencil that was 0.17 ___ long.
- Hold a "Measurement Scavenger Hunt" in the classroom. Students should estimate the length of objects first, and then measure for accuracy.
- Ask students: About how many cars, bumper to bumper, would there be in a line of cars a kilometre long? Explain how you determined your answer.

Possible Models: rulers, metre sticks, base ten blocks, measuring tapes, Cuisenaire® rods, trundle wheels

SCO: **SS2: Demonstrate an understanding of measuring length (mm and km) by:**
- **selecting and justifying referents for the units mm and km.**
- **modelling and describing the relationship between mm and cm units, and between mm and m units.**
- **modelling and describing the relationship between m and km units.**

[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Have students generalize measurement relationships between millimetres, centimetres, metres and kilometres from explorations using concrete materials (e.g., 10 mm = 1 cm; 0.01 m = 1 cm).
- Ask students to provide examples of what they would measure in millimetres (or kilometres). Why is this unit useful?
- Have students provide examples of situations that are relevant to their life, family, or community in which linear measurements would be made and identify the standard unit (mm, cm, m or km) that would be used for that measurement and justify the choice (e.g., heights of people, length of the bus).
- Have students use metric units to fill in the blanks below in as many ways as possible.
  1000 ____ = 1 ___ .
- Ask students to draw, construct, or physically act out a representation of a given linear measurement.
- Have students pose and solve problems that involve hands-on linear measurements using either referents or standard units.
- Tell students that a grasshopper hopped 1524 mm. Have them write the distance in metres.
- Have students look around the classroom and choose one object and estimate their measurement. Ask what referent they used to determine their measurements.
- Ask students: Which distance would you rather walk: 600 m or 6 km? Explain your choice.
- Have students measure the length of their desks in cm. Then ask them to measure in mm. Ask: which unit was most appropriate? Why?
- Ask students: What is something that you would measure in millimetres (kilometres)? Explain why you would use this unit. Why are these useful units?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS3: Demonstrate an understanding of volume by:**
- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
	SS3 Demonstrate an understanding of volume by: • selecting and justifying referents for cm^3 or m^3 units • estimating volume by using referents for cm^3 or m^3 • measuring and recording volume (cm^3 or m^3) • constructing rectangular prisms for a given volume.	**SS3** Develop and apply a formula for determining the: • perimeter of polygons • area of rectangles • volume of right rectangular prisms.

ELABORATION

Guiding Questions:

• What do I want my students to learn?

• What do I want my students to understand and be able to do?

Volume and **capacity** are both terms used for measuring the size of three-dimensional regions. Although these two concepts are related, we will concentrate on volume for this specific outcome. Volume typically refers to the amount of space that an object takes up. Volume is measured with cubic centimetres and cubic metres (Van de Walle & Lovin, vol. 2, 2006; p. 265). Volume can also be used to refer to the capacity of a container.

Students should develop personal referents for cubic centimetres and cubic metres. The use of personal referents helps students establish the relationships between the units. Students should realize that a cubic centimetre is the size of a cube 1 cm on each edge and a cubic metre the size of a cube 1 m on each edge. Being able to estimate the volume of various containers and then to measure in the appropriate unit is important as students begin to construct rectangular prisms of various sizes. By building a cubic metre with metre sticks, they will to have a good referent of 1 m^3. Students could then explore how many cubic centimetres it would take to equal the cubic metre (1 000 000 cm^3 = 1 m^3).

Students should develop a sense of which volume or capacity unit is more appropriate to use in various circumstances.

SCO: **SS3: Demonstrate an understanding of volume by:**

- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify the cube as the most efficient unit for measuring volume and explain why.
- Provide a referent for a cubic centimetre and explain the choice.
- Provide a referent for a cubic metre and explain the choice.
- Determine which standard cubic unit is represented by a given referent.
- Estimate the volume of a given 3-D object using personal referents.
- Determine the volume of a given 3-D object using manipulatives and explain the strategy.
- Construct a rectangular prism for a given volume.
- Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume.

SCO: **SS3: Demonstrate an understanding of volume by:**

- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students discuss a variety of situations where they need to choose the unit of measurement that would be used for each. Have them compare their answers and defend their choices (e.g., the unit of measure to find the volume of the paper clip box, a box of chocolates, the volume of a crate that would be used to transport a bicycle, a car, a dog).
- Provide frequent opportunities for students to construct different rectangular prisms and to discuss the volume of each solid.
- Have students find appropriate personal referents for cm^3 and m^3.

Suggested Activities

- Measure the volume of small rectangular prisms by counting the number of centimetre cubes it takes to build a duplicate of it.
- Use base ten blocks or linking cubes to build several different structures each with a set volume. Discuss the different dimensions of the rectangular prisms.
- Provide students with a pair of small boxes, one cube, and a ruler. Have students estimate which box has the greater volume and then determine how many cubes would be needed to fill each box. Students should use words, drawings and numbers to explain their conclusions.
- Have students build a cubic metre using metre sticks or other materials. Keep a model to use as a referent for m^3.
- Have students research the volume of moving trucks. Ask what is a reasonable estimate for the volume of all the furniture in a school or in a house?
- Give students 24 cubes. Have them build a rectangular prism using all 24 cubes so the volume equals 24 cm^3. Have them explore how many different rectangular prisms they can construct. Other volumes such as 16, 20, or 36 could also be investigated.

Possible Models: base ten blocks, linking cubes, rulers, measuring tapes, metre sticks, various models of rectangular prisms

SCO: **SS3: Demonstrate an understanding of volume by:**
- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

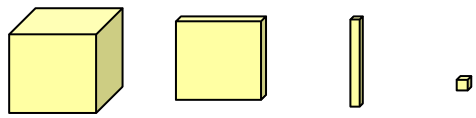
Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students to calculate the volume of each size of base-ten blocks.



- Ask students to estimate the volume of the classroom in cubic metres and give an explanation as to how the estimate was determined.
- Have students identify a 3-D object that could be measured in cubic centimetres and a 3-D object that would be measured in cubic metres and explain.
- Tell the student that you need a box with a volume of 400 cubic centimetres to hold a gift you have purchased. What might that gift be?
- Have students describe how area is different from volume.
- Give students the volume of a rectangular prism and have them construct it using centimetre cubes.
- Ask students to describe the strategy they would use to estimate the volume of certain common rectangular prisms such as lunchboxes, pasta boxes, tissue boxes, etc.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS4: Demonstrate an understanding of capacity by:**
- **describing the relationship between mL and L**
- **selecting and justifying referents for mL or L units**
- **estimating capacity by using referents for mL or L**
- **measuring and recording capacity (mL or L).**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

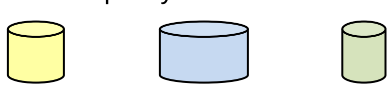
Grade Four	Grade Five	Grade Six
	SS4 Demonstrate an understanding of capacity by: • describing the relationship between mL and L • selecting and justifying referents for mL or L units • estimating capacity by using referents for mL or L • measuring and recording capacity (mL or L).	**SS3** Develop and apply a formula for determining the: perimeter of polygons; area of rectangles; volume of right rectangular prisms.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Volume and **capacity** are both terms for measuring the size of three-dimensional regions. Although these two concepts are related, this specific outcome is focused on capacity. It is useful for students to recognize the difference between volume (the amount of space occupied by a three-dimensional object) and capacity (the amount a container is capable of holding). Though this distinction is not critical, since the term "volume" is sometimes used to refer to the capacity of a container. Capacity units introduced in Grade 5 are millilitres (mL) and litres (L). Capacity units are usually associated with measures of liquid (e.g., milk, juice, medicine, and gasoline).

Since students have not had previous experience with capacity, investigation of capacity should begin with direct comparison and non-standard units. Give students containers of different sizes and shapes and have them order these from largest capacity to smallest capacity by comparing them or using the same small container to fill each. The investigation should next move to the introduction of standard measures. Using a variety of containers can help children see that container shapes can vary but the capacity may remain the same. Students should have the opportunity to compare containers that differ by only one dimension (e.g., the height of the containers is all the same, but the base is different sizes) to explore how this affects the capacity of the containers.



Students should develop personal referents for these units. The use of personal referents helps students establish the relationships between the units. They may think of common items such as a juice box or a milk container or use base ten blocks (e.g., the small cube in the base ten blocks is 1 cm^3 and would hold 1 mL and the large cube is 10 cm × 10 cm × 10 cm and would hold 1 L).

It is important for students to estimate capacities and to have a sense of which capacity unit (millilitre or litre) is the more appropriate to use in various circumstances.

SCO: **SS4: Demonstrate an understanding of capacity by:**
- **describing the relationship between mL and L**
- **selecting and justifying referents for mL or L units**
- **estimating capacity by using referents for mL or L**
- **measuring and recording capacity (mL or L).**

[C, CN, ME, PS, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Demonstrate that 1000 millilitres is equivalent to 1 litre by filling a 1 litre container using a combination of smaller containers.
- Provide a referent for a litre and explain the choice.
- Provide a referent for a millilitre and explain the choice.
- Determine which capacity unit is represented by a given referent.
- Estimate the capacity of a given container using personal referents.
- Determine the capacity of a given container using materials that take the shape of the inside of the container, e.g., a liquid, rice, sand, beads, and explain the strategy.

SCO: **SS4: Demonstrate an understanding of capacity by:**
- **describing the relationship between mL and L**
- **selecting and justifying referents for mL or L units**
- **estimating capacity by using referents for mL or L**
- **measuring and recording capacity (mL or L).**

[C, CN, ME, PS, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students discuss a variety of situations where they need to choose the unit of measurement that would be used for each. Have students compare their answers and defend their choices. For example, the unit of measure to find the capacity of cough syrup bottles, water bottles, juice boxes, yogurt containers, bathtub, gas tank, etc.
- Invite groups of students to investigate the capacities of various beverage containers to determine which size container is found most often.
- Provide ample opportunity for students to measure the capacity of different shaped and sized containers. Have students predict which unit of measure will be used.
- Have students find appropriate personal referents for litres and millilitres. They might look for containers at home and bring in empty samples.

Possible Referents	
Litres	**Millilitres**
Water bottle	Eye dropper
Milk jug	Spoon
Liquid detergent	Small yogurt container

Suggested Activities

- Have students measure the capacity of several different containers and record the most common capacities.
- Provide students with a pair of containers and ask them to predict which has the largest capacity (which holds more). Have them verify their predictions.
- Have students compare several cereal bowls to see how much a typical bowl can hold.
- Have students estimate the number of beans to fill a litre container and then check their estimation.
- Have students suggest containers for fixed capacities. (e.g., What kind of containers would hold 500 mL,1 L, or 250 mL?)

Possible Models: measuring cups and spoons (metric), various containers, base ten blocks

SCO: **SS4: Demonstrate an understanding of capacity by:**
- **describing the relationship between mL and L**
- **selecting and justifying referents for mL or L units**
- **estimating capacity by using referents for mL or L**
- **measuring and recording capacity (mL or L).**

[C, CN, ME, PS, R, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

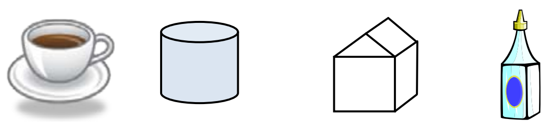
Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask students which capacity unit they would use to measure the capacity of the following and justify their choices.
  - swimming pool
  - coffee mug
  - cough medicine
  - bathtub
  - juice glass
- Tell students that a container holds 1.5 L. Ask if it is large enough to make a jug of orange juice, if the concentrate is 355 mL and you have to use the concentrate can to add three full cans of water.
- Ask the student how he/she could use a 1 L milk carton to estimate 750 mL of water.
- Provide students with 3 or 4 different containers and content to measure (liquid, sand, rice). Have students determine the capacities of the containers and explain their strategy.
- Have students describe their personal referent for millilitres and litres. Explain why each of these were selected
- Ask students to describe the strategy they would use to estimate the capacity of certain common containers such as water bottles, bathtub, various milk containers, etc.
- Provide a number of small containers that have labels showing the capacity of each in millilitres. Ask students to determine combinations of containers that would fill a 1 litre container.



150 mL 275 mL 200 mL 125 mL

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:**
- **parallel**
- **intersecting**
- **perpendicular**
- **vertical or horizontal.**

[C, CN, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
SS4 Describe and construct rectangular and triangular prisms.	**SS5** Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are: • parallel • intersecting • perpendicular • vertical or horizontal.	**SS4** Construct and compare triangles, including: scalene; isosceles; equilateral; right; obtuse; acute in different orientations.

ELABORATION

Guiding Questions:

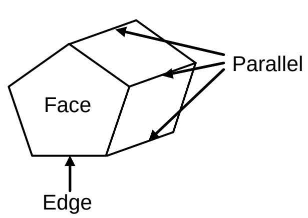
• What do I want my students to learn?

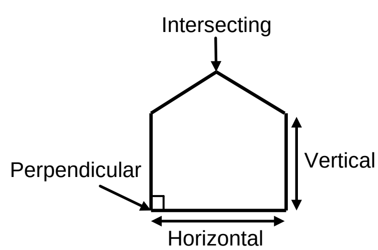
• What do I want my students to understand and be able to do?

There is a gradual progression from identifying and describing two- and three-dimensional objects in students' own words to identifying and describing them in the formal language of geometry. It is important that students become familiar with the vocabulary associated with describing the **attributes** of 2-D shapes and 3-D objects such as **parallel**, **intersecting**, **perpendicular**, **vertical** and **horizontal**.

Lines in the same plane can be parallel or they can intersect.
- Parallel lines never meet since they remain a constant distance apart.
- Intersecting lines meet at a single point.
- Perpendicular lines are intersecting lines that meet at a square corner.

Lines can also be vertical or horizontal.
- Vertical lines are up and down and are perpendicular to the horizon
- Horizontal lines are parallel to the horizon.





Students will also be expected to compare and describe 2-D shapes by relating their attributes and will also be expected to compare and describe 3-D objects in the same way. When given a set of attributes, students should be able to construct or draw the 2-D shape or 3-D object that corresponds to the description.

SCO: **SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:**
- **parallel**
- **intersecting**
- **perpendicular**
- **vertical or horizontal.**

[C, CN, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify parallel, intersecting, perpendicular, vertical and horizontal edges and faces on 3-D objects.
- Identify parallel, intersecting, perpendicular, vertical and horizontal sides on 2-D shapes.
- Provide examples from the environment that show parallel, intersecting, perpendicular, vertical and horizontal line segments.
- Find examples of edges, faces and sides that are parallel, intersecting, perpendicular, vertical and horizontal in print and electronic media, such as newspapers, magazines and the Internet.
- Draw 2-D shapes or 3-D objects that have edges, faces and sides that are parallel, intersecting, perpendicular, vertical or horizontal.
- Describe the faces and edges of a given 3-D object using terms, such as parallel, intersecting, perpendicular, vertical or horizontal.
- Describe the sides of a given 2-D shape using terms, such as parallel, intersecting, perpendicular, vertical or horizontal.

SCO: **SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:**
- **parallel**
- **intersecting**
- **perpendicular**
- **vertical or horizontal.**

[C, CN, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies
Consider the following strategies when planning lessons:
- Provide opportunities for students to manipulate 2-D shapes and 3-D objects and become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects such as parallel, intersecting, perpendicular, vertical and horizontal.
- Encourage students to use the formal language of geometry as they describe the attributes of classroom or real-life 2-D shapes and 3-D objects (e.g., the opposite walls in the classroom are parallel).
- Use pattern blocks, ask students to categorize sets of lines as parallel, intersecting, perpendicular, vertical or horizontal. Stack pattern blocks to build prisms. Pattern blocks will form triangular prisms, rectangular prisms, trapezoidal prisms, rhombus prisms, and hexagonal prisms.
- Go on a walk to look at 2-D shapes and 3-D objects in the environment. Have students discuss the attributes of shapes and objects in their environment using geometric language.

Suggested Activities
- Have students draw and construct on a geoboard 2-D shapes with specific attributes (e.g., construct a shape with a pair of parallel sides).
- Have students draw or construct (with toothpicks and marshmallows) 3-D objects and have them describe their attributes.
- Have students sort 2-D shapes and 3-D objects according to their attributes and justify their sorting scheme.
- Compare and describe the faces and edges of two prisms or two pyramids with different bases (e.g., triangular prisms and rectangular prisms.

Possible Models: geoboards, pattern blocks, geometric solids, grid paper, toothpicks and marshmallows, Geo-strips

SCO: **SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:**
- **parallel**
- **intersecting**
- **perpendicular**
- **vertical or horizontal.**

[C, CN, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Use a variety of geometric solids. Have students identify parallel, intersecting, and perpendicular edges.
- Provide students with several different 2-D shapes and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.
- Provide students with several different 3-D objects and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.
- Have students draw 2-D shapes and 3-D objects that satisfy a given set of attributes. For example, draw a parallelogram with both parallel and perpendicular sides (rectangle).
- Complete a Venn or Carroll diagram focusing on the attributes of 2-D shapes and 3-D objects as shown below.

Attributes	2-D object	3-D shape
Has parallel sides, edges or faces		
Does not have any parallel sides, edges or faces		

FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS6: Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

according to their attributes.
[C, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

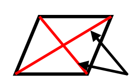
Grade Four	Grade Five	Grade Six
SS5 Demonstrate an understanding of line symmetry by: identifying symmetrical 2-D shapes; creating symmetrical 2-D shapes; drawing one or more lines of symmetry in a 2-D shape.	**SS6** Identify and sort quadrilaterals, including: • rectangles; squares • trapezoids • parallelograms • rhombuses according to their attributes.	**SS5** Describe and compare the sides and angles of regular and irregular polygons.

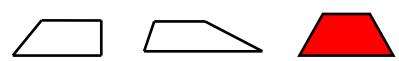
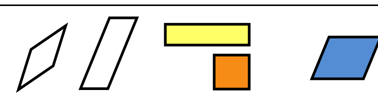
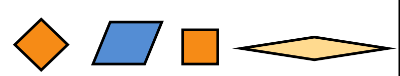
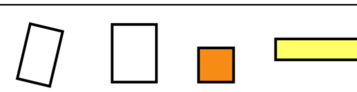
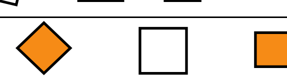
ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Quadrilaterals are polygons. All quadrilaterals have 4 straight sides and 4 angles. Although rectangles are the most common quadrilateral that you see in everyday life, students will soon discover that there are many classes of quadrilaterals (Small, 2008; p. 295). Students will be exploring the attributes of various quadrilaterals such as **trapezoids**, **parallelograms**, **rectangles**, **rhombuses** (or rhombi), and **squares**. They will compare the similarities and differences and sort them according to their attributes.

Common attributes may be lengths of sides, pairs of opposite parallel sides, lines of symmetry, and diagonals. All quadrilaterals have 2 diagonals (line that joins two non-adjacent vertices). It is important for students to recognize that some quadrilaterals can be classified in more than one category. For example, a square is also a rectangle and a parallelogram.


2 diagonals

Quadrilateral	Attributes	Examples
Trapezoid*	One pair of parallel sides Note: an isosceles trapezoid has a pair of opposite sides that are congruent (red example)	
Parallelogram	Two pairs of parallel sides Opposite sides are equal Opposite angles are equal	
Rhombus	A parallelogram with all sides congruent and equal opposite angles	
Rectangle	A parallelogram with four right angles (2 pairs of parallel sides; opposite sides are equal)	
Square	A parallelogram with four right angles and all sides congruent	

* A trapezoid is defined differently in different countries, but it is not the same as a trapezium.

SCO: SS6: Identify and sort quadrilaterals, including:
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

according to their attributes.
[C, R, V]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify and describe the characteristics of a pre-sorted set of quadrilaterals.
- Sort a given set of quadrilaterals and explain the sorting rule.
- Sort a given set of quadrilaterals according to the lengths of the sides.
- Sort a given set of quadrilaterals according to whether or not opposite sides are parallel.

SCO: **SS6: Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

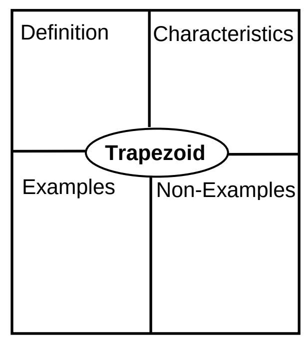
according to their attributes.
[C, R, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies
Consider the following strategies when planning lessons:
- Use models, drawings and real life examples of quadrilaterals, to identify and describe the characteristics of each and classify them. Have students explain their classification system.
- Provide the students with a template for the Frayer Model and have them fill in the sections individually to demonstrate their understanding of a geometric concept such a trapezoid.



Suggested Activities
- Have students go on a "Quadrilateral Scavenger Hunt". Have them sort their quadrilaterals with similar attributes and explain their rules for sorting.
- Have students prepare property lists with headings: sides, parallel, perpendicular, symmetries. Using a collection of quadrilaterals, have students describe the shapes using language such as: opposite sides equal, all sides equal, no sides equal or 2 pairs of parallel sides, one pair of parallel sides, no sides parallel.
- Provide students with a list of attributes and have them construct a quadrilateral that has the set of attributes. Have students share and compare with the class.
- Prepare a "Guess What Quadrilateral I Am?" game with clues about their attributes. The questions must have a "yes" or "no" answer (e.g., are opposite sides the same length?).

Possible Models: geometric solids, 2-D shapes, geoboards, pattern blocks, grid paper, Geo-strips

SCO: **SS6: Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

according to their attributes.
[C, R, V]

ASSESSMENT STRATEGIES

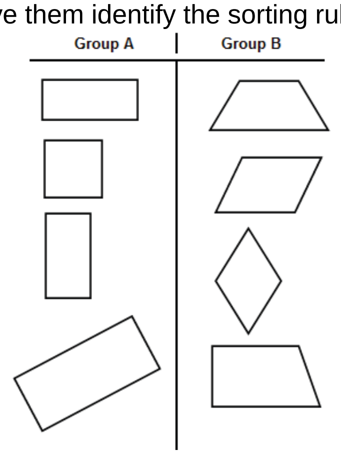
Look back at what you determined as acceptable evidence.

Guiding Questions
• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Provide students with several different quadrilaterals to sort and have them justify their sorting rule. Have them sort the shapes a different way and describe their sorting rule.
- Have students draw quadrilaterals that satisfy a given set of attributes. Be sure to include in the drawings the lengths of the sides and whether or not the opposite sides are parallel. Once they have the quadrilateral drawn, they should be able to identify the shape. For example:
  - a 2-D shape with four straight sides of equal length and four right angles;
  - a 2-D shape with four straight sides and four right-angles. One pair of sides is longer than the other.
  - a 2-D shape with four straight sides. One pair of sides is parallel with one side longer than the other.
- Provide students with a pre-sorted set of quadrilaterals and have them identify the sorting rule.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.**
[C, CN, T, V]
SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.
[C, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

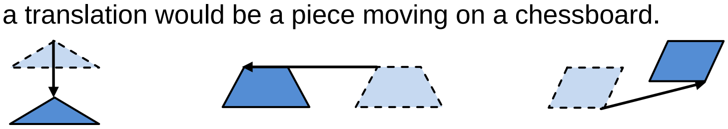
Grade Four	Grade Five	Grade Six
SS5 Demonstrate an understanding of line symmetry by: identifying symmetrical 2-D shapes; creating symmetrical 2-D shapes; drawing one or more lines of symmetry in a 2-D shape.	**SS7** Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. **SS8** Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**SS6** Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. **SS7** Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.

ELABORATION

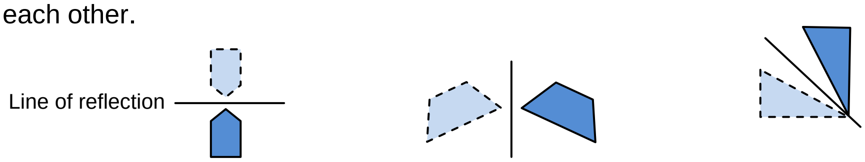
Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

There are three transformations that change the location of an object in space, or the direction, in which it faces, but not its size or shape. The three types of transformations are: **translations**, **reflections** and **rotations**. These transformations result in images that are congruent to the original object. Students are expected to identify and perform these three types of transformations.

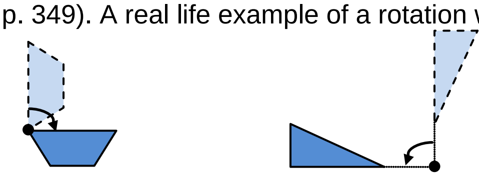
Translations move a shape left, right, up, down or diagonally without changing its orientation in any way. A real life example of a translation would be a piece moving on a chessboard.



Reflections can be thought of as the result of picking up a shape and turning it over. The reflection image is the mirror image of the original shape. A real life example of a reflection could be a pair of gloves beside each other.



Rotations move a shape around a **turn centre**. When students first start working with rotations they identify the amount of rotation as fractions of a circle (e.g., a **quarter turn**, **half turn** and **three-quarter turn**). Students are also expected to identify the direction of the rotation; **clockwise** and **counter-clockwise**. It is also important to identify the turn centre which could be outside of the shape or on the perimeter of the shape (Small, 2008; p. 349). A real life example of a rotation would be hands on a clock.



SCO: **SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.**
[C, CN, T, V]
SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.
[C, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

SS7

- Translate a given 2-D shape horizontally, vertically or diagonally, and describe the position and orientation of the image.
- Rotate a given 2-D shape about a point (turn centre), and describe the position and orientation of the image.
- Reflect a given 2-D shape in a line of reflection, and describe the position and orientation of the image.
- Perform a transformation of a given 2-D shape by following instructions.
- Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement.
- Draw a 2-D shape, rotate the shape and describe the direction of the turn (clockwise or counter-clockwise), the fraction of the turn and point of rotation.
- Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.
- Predict the result of a single transformation of a 2-D shape and verify the prediction.

SS8

- Provide an example of a translation, a rotation and a reflection.
- Identify a given transformation as a translation, rotation or reflection.
- Describe a given rotation by the direction of the turn (clockwise or counter-clockwise).

SCO: **SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.**
[C, CN, T, V]
SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.
[C, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students many opportunities to concretely translate a given 2-D shape using a geoboard and grid paper by identifying the direction and the magnitude of the translation.
- Provide students many opportunities to concretely reflect a given 2-D shape using a Mira® and grid paper ensure that a line of reflection is identified as well as the distance from the line of reflection is described.
- Provide students many opportunities to concretely rotate a given 2-D shape using a geoboard and grid paper ensure that a point of rotation is identified and that the fraction of the turn is described.
- Provide students with samples of wallpaper and have them explore various types of transformations.
- Explore this concept in other curricular areas such as art and physical education. Have students create their own wallpaper patterns using different transformations or act out a transformation in the gym.
- Have students discuss their predictions prior to performing a given transformation to a shape.

Suggested Activities

- Have students describe the direction as well as the magnitude of a given translation.
- Have students determine which transformation was performed on a given shape.
- Have students draw a shape and practice the different types of transformations using their shape. The transformations could be drawn on grid paper.
- Have students draw a shape, perform a transformation of their choice, draw the transformation on grid paper and have a partner describe the transformation that was performed.
- Ask students to perform a translation given the direction and magnitude of the movement.
- Ask students to perform a reflection given the line of reflection and the distance from the line of reflection.
- Ask students to perform a rotation given the direction of the turn (clockwise or counter-clockwise), the fraction of the turn and the turn centre (point of rotation). Include different turn centres.
- Ask students to create a shape on the geoboard, perform a transformation of their choice, and describe the transformation that was performed.
- Have students respond in their journal to the following prompts:
  - Explain using words and pictures if a translation can ever look like a reflection.
  - Explain using words and pictures how you know if a figure and its image show a reflection, translation, or rotation.

Possible Models: geoboards, grid paper, pattern blocks, tracing paper, Miras®

SCO: **SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.**
[C, CN, T, V]
SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.
[C, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Provide students with diagrams of different transformations and have them label each diagram with the type of transformation the diagram showed.
- Provide diagrams of rotations and ask, “Which picture shows a quarter turn? Half turn? Three-quarter turn.” Have students identify the turn centre of the rotation.
- Provide a 2-D shape and have students show a rotation, reflection, or translation on grid paper of that shape.
- Have students draw a shape, translate it, and then describe and explain the direction and the magnitude of the translation.
- Ask students to use their hands to demonstrate the three different transformations.
- Ask students to explain the differences and similarities among the three different transformations.
- Explain using words and pictures if a translation can ever look like a reflection.
- Explain using words and pictures how you know if a figure and its image show a translation, reflection, or rotation.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP1: Differentiate between first-hand and second-hand data.**
[C, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
SP1 Demonstrate an understanding of many-to-one correspondence.	**SP1** Differentiate between first-hand and second-hand data.	**SP2** Select, justify and use appropriate methods of collecting data, including: questionnaires; experiments; databases; electronic media.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students are familiar with collecting and organizing data from previous grades. They will learn about **first-hand data**, that they collect themselves, and **second-hand data** that other people have collected. The focus will be on comparing the collection methods and communicating results.

First-hand data: First-hand data is collected by the researcher (in this case the students) and is best used when they are looking for answers to questions about people, places or objects found in their everyday lives. First-hand data is required when this information is not readily available from existing respectable sources. It is also used when data is limited or when students are just beginning to learn about data. Collecting first-hand data can be done using a variety of methods such as interviews, surveys, experiments and observations. Students will need to determine what data they want to collect, gather the data, and then analyze it using **reasoning** to draw conclusions.

Second-hand data: Second-hand data is data that has been collected by someone else. It can be found in print and electronic media. Students will need to create appropriate questions that can be answered using second-hand data, and then use that data to communicate different conclusions.

This outcome provides an opportunity for students to work with large numbers in context, such as comparing populations (SCO N1).

SCO: **SP1: Differentiate between first-hand and second-hand data.**
[C, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain the difference between first-hand and second-hand data.
- Formulate a question that can best be answered using first-hand data and explain why.
- Formulate a question that can best be answered using second-hand data and explain why.
- Find examples of second-hand data in print and electronic media, such as newspapers, magazines and the Internet.

SCO: **SP1: Differentiate between first-hand and second-hand data.**
[C, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Provide questions and have students determine how best to collect the data in order for students to recognize the difference between first-hand and second-hand data.
- Have students generate questions that can best be answered using first-hand data. Describe how that data could be collected.
- Have students generate questions that can best be answered using second-hand data. Describe how that data could be collected.
- Have students explore and discuss the relevance of sample size on first- and second-hand data.
- Provide students with examples of first-hand and second-hand data and ask them to identify the type of data. Encourage students to reflect on the meaning of first-hand and second-hand data and record this reflection in their math journals. Have a class discussion on the two types of data.

Suggested Activities

- Provide examples of data relevant to self, family, or community and categorize the data, with explanation, as first-hand or second-hand data.
- Have students formulate questions that can best be answered using first-hand data (e.g., “What game will we play at home tonight?”). Students should describe how this data could be collected (“I can survey everyone at home to find out what games everyone wants to play.”). Have students collect data to answer the question.
- Have students formulate a question related to self, family, or community, which can best be answered using second-hand data (e.g., “Which has the larger population: my community or my friend’s community?”). Students should describe how this data could be collected (e.g., find the data on the StatsCan website: http://www.statcan.gc.ca). Have students collect data to answer the question.
- Have students find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet, and compare different ways in which the data might be interpreted and used (e.g., statistics about health-related issues, sports data, or votes for favourite websites).

Possible Models: examples of data from print and electronic data

SCO: **SP1: Differentiate between first-hand and second-hand data.**
[C, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment
- Ask students to write a question about preferred types of books that can be answered using first-hand data and explain why. How and from whom would the data be collected?
- Ask students to write a question about the populations of the cities in New Brunswick, and have them explain why the question is best answered using second-hand data. Where can they find this data?
- Have students work in groups to generate questions for which the data would be collected first- and second-hand.
- Provide students with a set of data and have them generate questions that could be answered using it.
- Ask students to explain the difference between first-hand and second-hand data and give examples.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP2: Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.	**SP2** Construct and interpret double bar graphs to draw conclusions.	**SP1** Create, label and interpret line graphs to draw conclusions. **SP3** Graph collected data and analyze the graph to solve problems.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

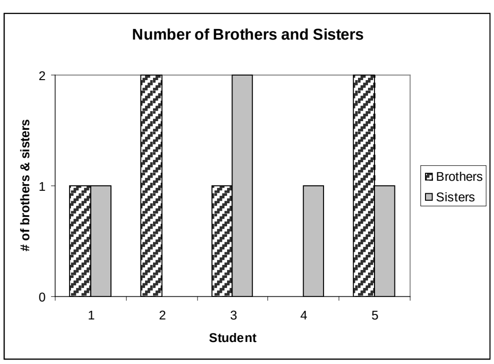
Students should be aware that sometimes when two pieces of data are collected about a certain population, it is desirable to display both sets of data side by side, using the same scale. For example, census data often shows male and female data separately for different years. This is usually done using a **double bar graph**. A **legend** is used to help the reader interpret a double bar graph. An example is presented below. Five students in the class have been asked how many brothers and sisters they have.

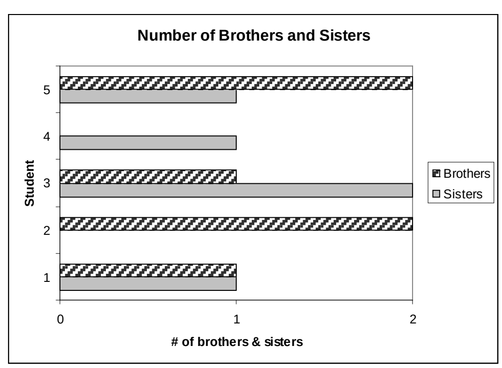
This type of graph allows students to be compared not only in terms of how many brothers they have, or how many sisters they have, but also to compare the number of brothers versus the number of sisters.

It is essential that students include **titles**, **horizontal** and **vertical axis** headings and **scale**, **legends** and **category labels** in the **legend**. The pairs of bars should be separated and the order of the colours must remain the same in the graph.
A common mistake made by students is to place the numbers on the scale in the space between lines rather than on the place where the line for the limit of that number would be (e.g., 1, 2, etc.).

	Brothers	Sisters
Student 1	1	1
Student 2	2	0
Student 3	1	2
Student 4	0	1
Student 5	2	1

The data may be displayed horizontally or vertically as shown below.





SCO: **SP2: Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

ACHIEVEMENT INDICATORS

Guiding Questions:

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Determine the attributes (title, axes, intervals and legend) of double bar graphs by comparing a given set of double bar graphs.
- Represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology.
- Draw conclusions from a given double bar graph to answer questions.
- Provide examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines and the Internet.
- Solve a given problem by constructing and interpreting a double bar graph.

SCO: **SP2: Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

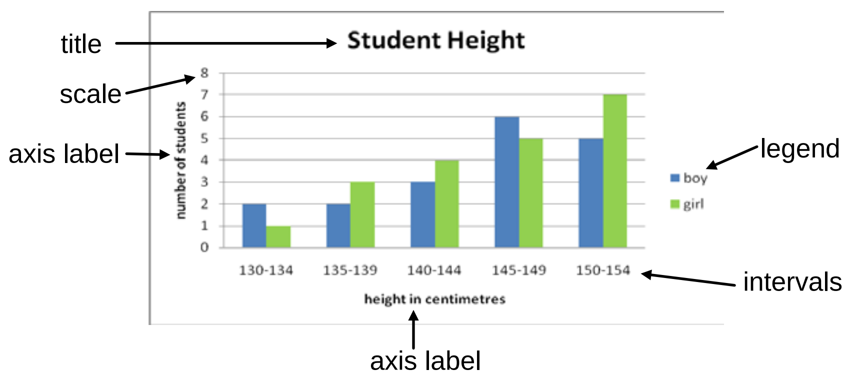
Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students determine when it is appropriate to display data in a double bar graph.
- Provide students with sets of data and have them determine appropriate scales.
- Provide students with two double bar graphs displaying the same data using a different scale, and have students determine which they prefer and why.
- Have students collect first-hand and second-hand data and create double bar graphs making sure to include appropriate title, axis labels, scale, and legend.
- Have students use second-hand data collected from sites, such as Statistics Canada that use large numbers to interpret the data on double bar graphs found on these sites (http://www.statcan.gc.ca and Census at School: www.censusatschool.ca).
- Have students interpret a given double bar graph to answer a set of questions.
- Have students generate sets of questions that can be answered by reading various double bar graphs.
- Have students compare data in the double bar graph within and among the pairs.

Suggested Activities

- Provide examples of double bar graphs from a variety of media sources, and ask students to bring in examples from similar sources.
- Have students examine double bar graph samples and determine the attributes (title, axes, legend, intervals). Ask them to compare and share the information displayed.
- Have students collect and graph first-hand data, such as girls' and boys' favourite activity in gym.
- Have students collect information on the length and mass of various animals and display the data in a double bar graph. Ask what conclusions they might draw.
- Have students create double bar graphs on subjects that are of personal interest, such as comparing hockey players' salaries from two different teams



Possible Models: double bar graph samples from various media sources, graph paper

SCO: **SP2: Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions

• What are the most appropriate methods and activities for assessing student learning?
• How will I align my assessment strategies with my teaching strategies?

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

Whole Class/Group/Individual Assessment

- Ask the student to describe some data that would be appropriate to display using a double bar graph.
- Have students generate a double bar graph from given sets of data without the use of technology. Rubrics might include appropriate scales and labeling as well as accuracy. Ensure that it includes a title, labels, scale, and a legend.
- Provide students with a double bar graph and have them identify the title, labels, scale, and legend. Ask them to describe why it is important to include each of these for a double bar graph.
- Have students construct a double bar graph to help them solve a real-world problem. Ask students to draw one conclusion based on their graph.
- Have students draw conclusions from a given double bar graph to answer questions.
  - What information does the graph show?
  - What kinds of data were collected?
  - How many pieces of data were involved?
  - What conclusions can be drawn based on this data?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

• What conclusions can be made from assessment information?
• How effective have instructional approaches been?
• What are the next steps in instruction?

SCO: **SP3: Describe the likelihood of a single outcome occurring using words, such as:**
- **impossible**
- **possible**
- **certain.**

SP4: Compare the likelihood of two possible outcomes occurring using words, such as:
- **less likely**
- **equally likely**
- **more likely.**

[C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Four	Grade Five	Grade Six
	SP3 Describe the likelihood of a single outcome occurring using words, such as: • impossible • possible • certain. **SP4** Compare the likelihood of two possible outcomes occurring using words, such as: • less likely • equally likely • more likely.	**SP4** Demonstrate an understanding of probability by: identifying all possible outcomes of a probability experiment; differentiating between experimental and theoretical probability; determining the theoretical probability of outcomes in a probability experiment; determining the experimental probability of outcomes in a probability experiment; comparing experimental results with the theoretical probability for an experiment.

ELABORATION

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

The occurrence of a future event can be characterized along a **continuum** from **impossible** to **certain**. The key idea to developing **chance** or **probability** on a continuum is to help children see that some events are more likely than others. Before students attempt to assign numeric probabilities to events, it is important that they have the basic idea that some events are certain to happen, some are certain not to happen or are impossible, and others have different chances of occurring that fall between these extremes (Van de Walle & Lovin, vol. 2, 2006, p. 340).

Students should be encouraged to use their **reasoning skills** to make predictions about outcomes, and to communicate the results using probability language. This introduction to the probability of an event gives students the opportunity to bring their own life experiences to the discussion.

Once students have mastered the concept of **likelihood** (probability) of a single outcome occurring, they can then begin to compare the likelihood of two outcomes occurring, using the comparative language **less likely**, **equally likely**, **more likely**.

Students will design and conduct probability experiments for the likelihood of single outcomes occurring, as well as a comparison of two outcomes. They will be expected to record the outcomes and explain the results.

SCO: **SP3: Describe the likelihood of a single outcome occurring using words, such as:**
- **impossible**
- **possible**
- **certain.**

SP4: Compare the likelihood of two possible outcomes occurring using words, such as:
- **less likely**
- **equally likely**
- **more likely.**

[C, CN, PS, R]

ACHIEVEMENT INDICATORS

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

SP3
- Provide examples of events that are impossible, possible or certain from personal contexts.
- Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible or certain.
- Plot the likelihood of a single outcome occurring in a probability experiment along a continuum.
- Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible or certain.
- Conduct a given probability experiment a number of times, record the outcomes and explain the results.

SP4
- Identify outcomes from a given probability experiment which are less likely, equally likely or more likely to occur than other outcomes.
- Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.
- Design and conduct a probability experiment in which one outcome is equally as likely to occur as the other outcome.
- Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.

SCO: **SP3: Describe the likelihood of a single outcome occurring using words, such as:**
- **impossible**
- **possible**
- **certain.**

SP4: Compare the likelihood of two possible outcomes occurring using words, such as:
- **less likely**
- **equally likely**
- **more likely.**

[C, CN, PS, R]

PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

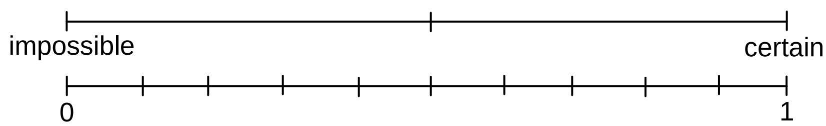
Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

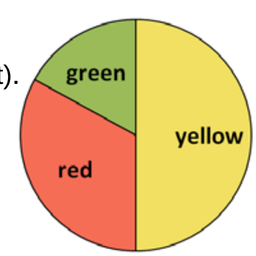
- Provide a list of single outcomes (events) ranging from impossible to certain and ask students to identify the likelihood of the event occurring using probability language.
- Have students plot outcomes on a probability continuum accompanied by an explanation as to their placement.



- Have students generate a list of events that would fall on the probability continuum (e.g., tomorrow is Friday; snow in August in New Brunswick).
- Have students conduct a given probability experiment in which the likelihood of a single outcome occurring is impossible, possible or certain, recording and explaining their results.
- Have students design and conduct probability experiments on a single outcome, recording and explaining their results.
- Give students frequent opportunities to identify outcomes from given probability experiments that are less likely, equally likely or more likely to occur than other outcomes.
- Use children's literature, such as *Cloudy with a Chance of Meatballs* by Judi Barrett to discuss how probability language is used in our daily lives.
- Have students design and conduct probability experiments in which one outcome is less likely, equally likely and more likely to occur than the other outcome.

Suggested Activities

- Have the student design experiments for which a certain outcome is impossible, possible and certain.
- Have the student design experiments with two possible outcomes in which one of the outcomes is less likely, equally likely or more likely to occur.
- Ask the student to design a spinner so that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow (see example at the right).
- Have students create their own probability events and have other classmates place them on the probability line.



Possible Models: number cubes (dice), spinners, colour tiles

SCO: **SP3: Describe the likelihood of a single outcome occurring using words, such as:**
- **impossible**
- **possible**
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SP4: Compare the likelihood of two possible outcomes occurring using words, such as:
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[C, CN, PS, R]

ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

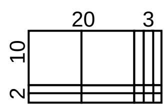
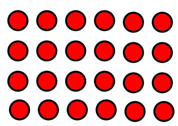
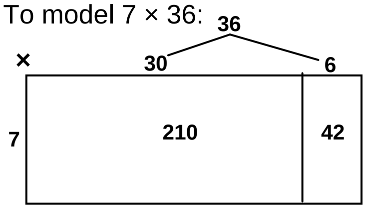
Whole Class/Group/Individual Assessment
- Tell the student he/she wins $1 if a spinner lands on red, and loses $1 if it lands on blue. Ask: How would you like the spinner to be designed?
- Ask the student to think of an event that is possible, but not very likely, and another event that is very likely, but might not happen.
- Write a journal describing events that are impossible, possible and certain in our everyday lives.
- Provide students with a number cube (die). Have students describe a result of rolling a number cube that is:
  - less likely to occur than another outcome (e.g., a number less than 3 vs. one greater than 3);
  - equally likely to occur (e.g., an even number vs. an odd number);
  - more likely to occur than another outcome (e.g., a number less than 5 vs. one greater than 5).
- Provide students a paper bag and 10 colour tiles: 6 red, 3 yellow, 1 blue. Have students design and conduct an experiment to determine the probability of choosing a red tile. Explain.
- Have students toss a coin twenty-five times and record their results in a chart. Flip the coin another twenty-five times and record the results. Explain the results.
- Provide students a paper bag and 20 colour tiles: 8 blue, 5 green, 5 blue, and 2 yellow. Have students design and conduct an experiment in which:
  - one outcome is less likely to occur than other outcomes;
  - outcomes are equally likely to occur;
  - one outcome is more likely to occur than other outcomes.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

GLOSSARY OF MODELS

This glossary is identical for all grade levels (kindergarten to grade 8). Most of the models have a variety of uses at different grade levels. More information as to which models can be used to develop specific curriculum outcomes is located on the *Instructional Strategies* section of each four-page spread in this curriculum document. The purpose of this glossary is to provide a visual of each model and a brief description of it.

Name	Picture	Description
Algebra tiles		• Sets include "X" tiles (rectangles), "X^2" tiles (large squares), and integer tiles (small squares). • All tiles have a different colour on each side to represent positive and negative. Typically the "X" tiles are green and white and the smaller squares are red and white. • Some sets also include "Y" sets of tiles which are a different colour and size than the "X" tiles.
Area Model	To model 12 × 23: 	• Use base ten blocks to represent the parts of each number that is being multiplied. • To find the answer for the example shown, students can add the various parts of the model: 200 + 30 + 40 + 6 = 276. • This model can also be used for fraction multiplication.
Arrays and Open Arrays	To model 4 × 6:  To model 7 × 36: 	• Use counters arranged in equal rows or columns or a Blackline Master with rows and columns of dots. • Helpful in developing understanding of multiplication facts. • Grids can also be used to model arrays. • Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.

Attribute Blocks		• Sets of blocks that vary in their attributes:
		  o 5 shapes circle, triangle, square, hexagon, rectangle
		  o 2 thicknesses
		  o 2 sizes
		  o 3 colours

Balance (pan or beam) scales		• Available in a variety of styles and precision. • Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units. • Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.

<table>
<tr><td>Base Ten Blocks</td><td></td><td>

- Include unit cubes, rods, flats, and large cubes.
- Available in a variety of colours and materials (plastic, wood, foam).
- Usually 3-D.

</td></tr>
<tr><td>Beam Balance</td><td colspan="2">☞ see Balance (pan or beam)</td></tr>
<tr><td>Carroll Diagram</td><td>Example:
<table>
<tr><td></td><td>1-digit</td><td>2-digit</td></tr>
<tr><td>Even</td><td>2, 4, 6, 8</td><td>26, 34</td></tr>
<tr><td>Odd</td><td>1, 3, 5, 7, 9</td><td>15, 21</td></tr>
</table></td><td>

- Used for classification of different attributes.
- The table shows the four possible combinations for the two attributes.
- Similar to a Venn Diagram

</td></tr>
<tr><td>Colour Tiles</td><td></td><td>

- Square tiles in 4 colours (red, yellow, green, blue).
- Available in a variety of materials (plastic, wood, foam).

</td></tr>
<tr><td>Counters (two colour)</td><td></td><td>

- Counters have a different colour on each side.
- Available in a variety of colour combinations, but usually are red & white or red & yellow.
- Available in different shapes (circles, squares, bean).

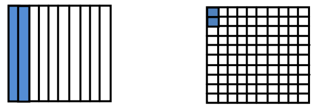
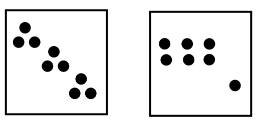
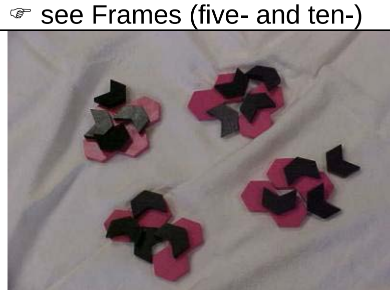
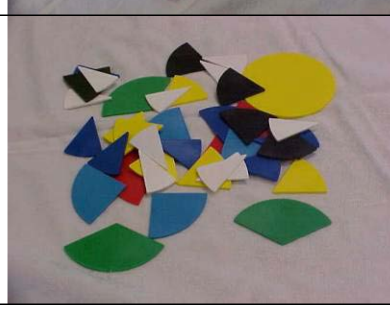
</td></tr>
<tr><td>Cubes (Linking)</td><td></td><td>

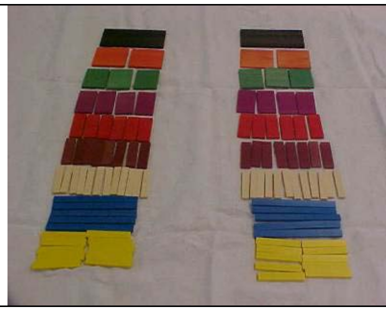
- Set of interlocking 2 cm cubes.
- Most connect on all sides.
- Available in a wide variety of colours (usually 10 colours in each set).
- Brand names include: Multilink, Hex-a-Link, Cube-A-Link.
- Some types only connect on two sides (brand name example: Unifix).

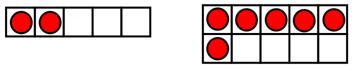
</td></tr>
<tr><td>Cuisenaire Rods®</td><td></td><td>

- Set includes 10 different colours of rods.
- Each colour represents a different length and can represent different number values or units of measurement.
- Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange).
- Available in plastic or wood.

</td></tr>
</table>

Decimal Squares®		• Tenths and hundredths grids that are manufactured with parts of the grids shaded. • Can substitute a Blackline Master and create your own class set.
Dice (Number Cubes)		• Standard type is a cube with numbers or dots from 1 to 6 (number cubes). • Cubes can have different symbols or words. • Also available in: o 4-sided (tetrahedral dice) o 8-sided (octahedral dice) o 10-sided (decahedra dice) o 12-sided, 20-sided, and higher o Place value dice
Dominoes		• Rectangular tiles divided in two-halves. • Each half shows a number of dots: 0 to 6 or 0 to 9. • Sets include tiles with all the possible number combinations for that set. • Double-six sets include 28 dominoes. • Double-nine sets include 56 dominoes.
Dot Cards		• Sets of cards that display different number of dots (1 to 10) in a variety of arrangements. • Available as free Blackline Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).
Double Number Line	☞ see Number lines (standard, open, and double)	
Five-frames	☞ see Frames (five- and ten-)	
Fraction Blocks		• Also known as Fraction Pattern blocks. • 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle. • Use with basic pattern blocks to help study a wider range of denominators and fraction computation.
Fraction Circles		• Sets can include these fraction pieces: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ • Each fraction graduation has its own colour. • It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).

<table>
<tr><td>Fraction Pieces</td><td></td><td>

Rectangular pieces that can be used to represent the following fractions:
$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$
Offers more flexibility as different pieces can be used to represent 1 whole.
Each fraction graduation has its own colour.
Sets available in different quantities of pieces.
</td></tr>
<tr><td>Frames
(five- and ten-)</td><td></td><td>

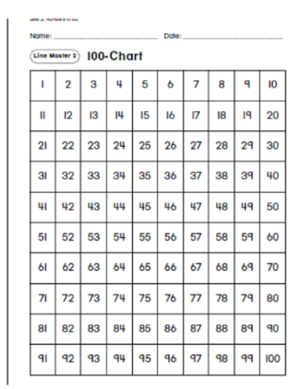
Available as a Blackline Master in many resources or you can create your own.
Use with any type of counter to fill in the frame as needed.
</td></tr>
<tr><td>Geoboards</td><td></td><td>

Available in a variety of sizes and styles.

5 × 5 pins
11 × 11 pins
Circular 24 pin
Isometric

Clear plastic models can be used by teachers and students on an overhead.
Some models can be linked to increase the size of the grid.
</td></tr>
<tr><td>Geometric Solids</td><td></td><td>

Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres.
The number of pieces in a set will vary.
Available in different materials (wood, plastic, foam) and different sizes.
</td></tr>
<tr><td>Geo-strips</td><td></td><td>

Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes.
Strips come in 5 different lengths. Each length is a different colour.
</td></tr>
<tr><td>Hundred Chart</td><td>
Name: Date:
Line Master 3 100-Chart
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<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
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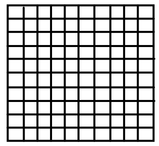
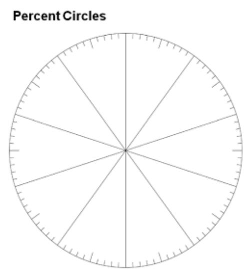
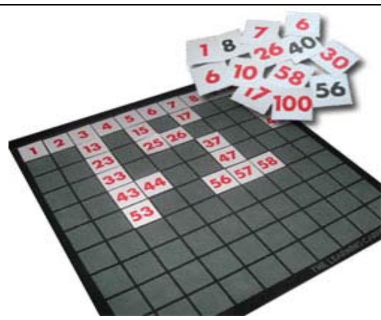
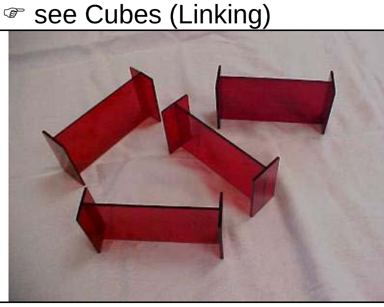
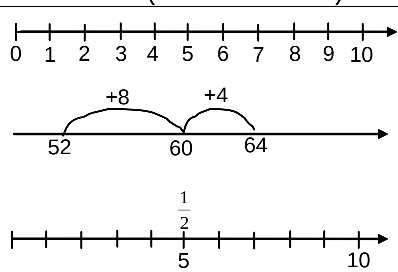
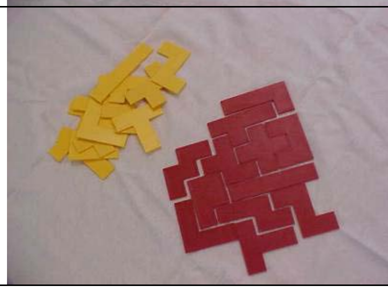
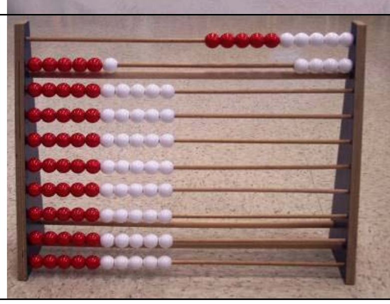
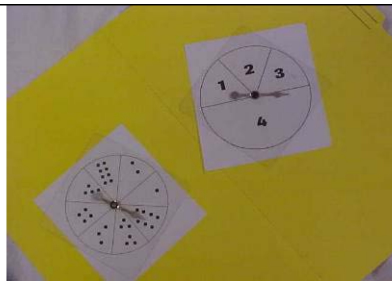
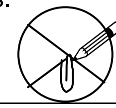
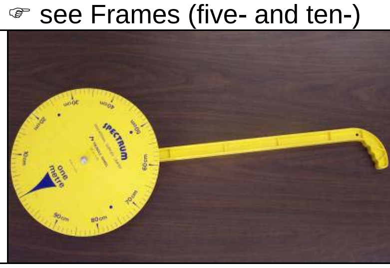
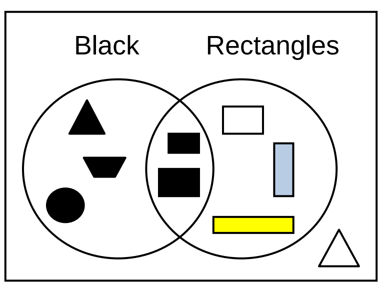
<tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr>
<tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr>
</table></td><td>

10 × 10 grid filled in with numbers 1-100 or 0 - 99.
Available as a Blackline Master in many resources or you can create your own.
Also available as wall charts or “Pocket” charts where cards with the numbers can be inserted or removed.
</td></tr>
</table>

Hundred Grid		• 10 × 10 grid. • Available as Blackline Master in many resources.
Hundredths Circle		• Circle divided into tenths and hundredths. • Also known as “percent circles”.
Learning Carpet®	 http://www.thelearningcarpet.ca	• 10 × 10 grid printed on a floor rug that is six feet square. • Number cards and other accessories are available to use with the carpet.
Linking Cubes	☞ see Cubes (Linking)	
Mira®		• Clear red plastic with a bevelled edge that projects reflected image on the other side. • Other brand names include: Reflect-View and Math-Vu™.
Number Cubes	☞ see Dice (Number Cubes)	
Number Lines (standard, open, and double)		• Number lines can begin at 0 or extend in both directions. • Open number lines do not include pre-marked numbers or divisions. Students place these as needed. • Double number lines have numbers written above and below the line to show equivalence.
Open Arrays	☞ see Arrays and Open Arrays	
Open Number Lines	☞ see Number Lines (standard, open, and double)	
Pan Balance	☞ see Balance (pan or beam)	

Pattern Blocks		• Standard set includes: Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms. • Available in a variety of materials (wood, plastic, foam).
Pentominoes		• Set includes 12 unique polygons. • Each is composed of 5 squares which share at least one side. • Available in 2-D and 3-D in a variety of colours.
Polydrons		• Geometric pieces snap together to build various geometric solids as well as their nets. • Pieces are available in a variety of shapes, colours, and sizes: Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons • Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.
Power Polygons™		• Set includes the 6 basic pattern block shapes plus 9 related shapes. • Shapes are identified by letter and colour.
Rekenrek		• Counting frame that has 10 beads on each bar: 5 white and 5 red. • Available with different number of bars (1, 2, or 10).

Spinners		• Create your own or use manufactured ones that are available in a wide variety: o number of sections; o colours or numbers; o different size sections; o blank. • Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins. 
Tangrams		• Set of 7 shapes (commonly plastic): o 2 large right-angle triangles o 1 medium right-angle triangle o 2 small right-angle triangles o 1 parallelogram o 1 square • 7-pieces form a square as well as a number of other shapes. • Templates also available to make sets.
Ten-frames	☞ see Frames (five- and ten-)	
Trundle Wheel		• Tool for measuring longer distances. • Each revolution equals 1 metre usually noted with a click.
Two Colour Counters	☞ see Counters (two colour)	
Venn Diagram		• Used for classification of different attributes. • Can be one, two, or three circles depending on the number of attributes being considered. • Attributes that are common to each group are placed in the interlocking section. • Attributes that don't belong are placed outside of the circle(s), but inside the rectangle. • Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted. • Similar to a Carroll Diagram.

List of Grade 5 Specific Curriculum Outcomes

<u>Number</u> (N)

1. Represent and describe whole numbers to 1 000 000.
2. Use estimation strategies, including: front-end rounding; compensation; compatible numbers in problem-solving contexts.
3. Apply mental mathematics strategies and number properties, such as: skip counting from a known fact; using doubling or halving; using patterns in the 9s facts; using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.
4. Apply mental mathematics strategies for multiplication, such as: annexing then adding zero; halving and doubling; using the distributive property.
5. Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.
6. Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.
7. Demonstrate an understanding of fractions by using concrete and pictorial representations to: create sets of equivalent fractions; compare fractions with like and unlike denominators.
8. Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.
9. Relate decimals to fractions (to thousandths).
10. Compare and order decimals (to thousandths), by using: benchmarks; place value; equivalent decimals.
11. Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

<u>Patterns & Relations</u> (PR)

(Patterns)

1. Determine the pattern rule to make predictions about subsequent terms (elements).

(Variables and Equations)

2. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

<u>Shape and Space</u> (SS)

(Measurement)

1. Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.
2. Demonstrate an understanding of measuring length (mm and km).
3. Demonstrate an understanding of volume by: selecting and justifying referents for cm^3 or m^3 units; estimating volume by using referents for cm^3 or m^3; measuring and recording volume (cm^3 or m^3); constructing rectangular prisms for a given volume.
4. Demonstrate an understanding of capacity by: describing the relationship between mL and L; selecting and justifying referents for mL or L units; estimating capacity by using referents for mL or L; measuring and recording capacity (mL or L).

(3-D Objects and 2-D Shapes)

5. Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are: parallel; intersecting; perpendicular; vertical; horizontal.
6. Identify and sort quadrilaterals, including: rectangles; squares; trapezoids; parallelograms; rhombuses according to their attributes.

(Transformations)

7. Perform a single transformation (translation, rotation or reflection) of a 2-D shape, (with and without technology) and draw and describe the image.
8. Identify a single transformation including a translation, a rotation and a reflection of 2-D shapes.

<u>Statistics and Probability</u> (SP)

(Data Analysis)

1. Differentiate between first-hand and second-hand data.
2. Construct and interpret double bar graphs to draw conclusions.

(Chance and Uncertainty)

3. Describe the likelihood of a single outcome occurring using words, such as: impossible; possible; certain.
4. Compare the likelihood of two possible outcomes occurring using words, such as: less likely; equally likely; more likely.

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